“The real wisdom is simply to recall that one cannot solve a problem without recognizing that it exists and that the ultimate form of silliness is to keep doing the same thing all the while expecting different outcomes.”

Jean Brunel

Nathan Sosner, Stanley Krasner, and Ted Pyne

The Tax Benefits of Relaxing the Long-Only Constraint: Do They Come from Character or Deferral?
The Tax Benefits of Relaxing the Long-Only Constraint: Do They Come from Character or Deferral?

NATHAN SOSNER, STANLEY KRASNER, AND TED PYNE

Prior literature shows that relaxing the long-only constraint significantly improves pretax implementation efficiency. More recently, Berkin and Luck [2010] and Sialm and Sosner [2018] found that relaxing the long-only constraint also enhances tax efficiency, in particular for tax-aware strategies. In this study, we propose a decomposition of the current year’s total tax benefit (or liability) of a strategy into what we define as character and deferral components. Our decomposition is mathematically straightforward, intuitive, and in our view helpful to taxable investors and their advisors seeking to understand and improve the after-tax performance of their investment portfolios. We use this decomposition to identify the source of tax benefits resulting from relaxation of the long-only constraint. Our methodology clearly shows which taxable investors would benefit the most from combining a limited amount of shorting with tax-aware rebalancing.

Numerous books and articles have advised taxable investors to optimize their investment portfolios for taxes. However, to our knowledge, our study is the first in this expansive literature to define the decomposition of tax benefits into character and deferral and explain its relevance for assessing the tax benefits of several well-known investment strategies.

There are two ways of achieving a tax benefit at the level of an overall investment portfolio held in a taxable account. First, an investor can favorably affect the character of realized capital gains and income at the overall portfolio level by tilting the balance of net realized gains in a given year from short-term to long-term and from ordinary income to qualified dividends and tax-exempt income. The benefit results from relaxation of the long-only constraint. Our methodology clearly shows which taxable investors would benefit the most from combining a limited amount of shorting with tax-aware rebalancing.

1 The argument in favor of implementation efficiency of relaxed-constraint strategies was made by Clarke, de Silva, and Sapra (2004); Jacobs and Levy (2006); Berger (2008); and Ang, Michalka, and Ross (2017). In addition, Jacobs and Levy (2007) dispelled a number of widespread misconceptions related to relaxed-constraint portfolio construction and the risks and costs associated with managing relaxed-constraint strategies.

2 A short and incomplete list of examples of such advice include Stein and Narasimhan (1999); Arnott, Berkin, and Ye (2001b); Brunel (2001, 2006); Rogers (2001, 2006); Stein (2001); Berkin and Ye (2003); Horvitz and Wilcoxon (2003); Quisenberry (2003); Stein and McIntire (2003; Paulson and Tavel (2005); Wilcoxon, Horvitz, and deBartolomeo (2006); Horan and Adler (2009); Jennings et al. (2011); Kim, Dougherty, and Klein (2011); Israel and Moskowitz (2012); Bouchey, Santodomingo, and Sireklove (2015); Bouchey, Brunel, and Li (2016), Lucas and Sanz (2016); and Bouchey and Pritamani (2017).

3 Technically speaking, capital gains (and losses) and ordinary income (and deductions) are the only two true categories of character. Long-term and short-term capital gains are subcategories within the capital gains character. However, they are often referred to as having a different character because under the current US tax law they are subject to different tax rates.
from a lower amount of highly taxed short-term capital gains and income as a fraction of all realized gains and income. Second, at the overall portfolio level, an investor can defer the realization of capital gains to future years and benefit from a reduction in the current year’s taxable gains. In this case, the benefit arises from a lower amount of realized gains as a fraction of the total investment profits—realized and unrealized. Whereas the character benefit is permanent (i.e., tax liabilities are reduced permanently by paying tax at a lower rate), the deferral benefit or liability is temporal: Barring a tax-exempt portfolio liquidation resulting from a donation to charity or step-up in cost basis at death, an increase in current unrealized gains (current deferral benefit) leads to higher liquidation taxes, whereas a decrease in current unrealized gains (current deferral liability) leads to lower liquidation taxes. Despite its temporal nature, current deferral benefit adds real value because it allows the investor’s wealth to appreciate (compound) at a faster rate, even after adjusting for a higher future liquidation tax resulting from gain deferral. 4

At the level of a single strategy within an overall investment portfolio, a character benefit occurs when the strategy allocates gains and income in low taxed characters, such as long-term capital gains and qualified dividends, and a matching amount of losses and deductions in highly taxed characters, such as short-term capital losses and ordinary deductions. This is because short-term losses offset short-term gains before offsetting any long-term gains. Thus, a strategy realizing a similar amount of long-term gains and short-term losses tilts the balance of net realized gains in a given year from short term to long term at the overall portfolio level. This is consistent with the definition of character benefit at the overall portfolio level defined in the previous paragraph. (Note that here and throughout the article we assume that a strategy is managed in a separately managed account or in a limited partnership—both vehicles allow pass-through of strategy positions’ realized losses and deductions to the investor.) Similarly, at the single strategy level, a deferral benefit occurs when the strategy allocates losses and deductions in excess of gains and income. A strategy realizing deferral benefit is reducing the net

realized gains and income of the overall investment portfolio in the current year, which is again consistent with the definition of deferral benefit in the previous paragraph. 5 Continuing with this definition, a strategy that adds realized gains and/or income to the investment portfolio realizes a deferral liability. The flip side of what we define as deferral benefit (liability) is an increase (decrease) in unrealized gains and thus an expected future tax liability (benefit). 6

Relaxed-constraint strategies are particularly interesting for this type of character-deferral analysis because they combine features of traditional active long-only asset management and alternative long–short hedge fund investing. Similar to active long-only, they seek to provide benchmark exposure and a higher-than-benchmark return and thus can be viewed as a substitute for traditional active management. Similar to long–short hedge fund strategies, they use leverage and shorting to achieve active return. Through the lens of the character-deferral decomposition, we explore which types of tax benefits (and liabilities) relaxed-constraint strategies inherit from their long-only and long–short counterparts. Our empirical evidence shows that for tax-aware strategies, relaxing the long-only constraint results in a large increase in tax benefits, in particular character benefit. We thus conclude that tax-aware relaxed-constraint strategies are more attractive than tax-aware long-only strategies to taxable investors. In the final section, we discuss caveats that might affect this conclusion.

Before we present our character-deferral tax benefit decomposition, in the next section, we introduce a tax-aware relaxed-constraint equity strategy and compare it to other types of tax-aware equity strategies

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4 As will be seen later, our after-tax return calculation methodology explicitly accounts for the liquidation tax costs (benefits) of an increase in unrealized gains (losses).

5 An important caveat is that the tax benefits of a strategy arise when its losses offset gains from other investments in the investor’s portfolio. This assumption will be employed throughout the article.

6 A comparison between, for example, an equity exchange-traded fund (ETF) and a long-dated equity exchange-traded note (ETN) might help clarify the concept of deferral liability. Even a tax-efficient equity ETF would still distribute dividend income paid out by the underlying stocks and thus would have a deferral liability in any given year. A long-dated ETN only has one payment at a distant future maturity date and, as a result, has no deferral liability in the current year but a larger expected tax liability in a future year when it matures. Under our definition, the ETN does not have a deferral benefit, though, because it does not offset current gains or income of other strategies in the investment portfolio.
described in prior literature. Readers familiar with the topic may skip the next section.

A BRIEF OVERVIEW OF TAX-AWARE STRATEGIES

Relaxing the long-only constraint improves the implementation efficiency of actively managed portfolios: Although long-only strategies are limited in their ability to express negative views on stocks, relaxed-constraint strategies can implement negative views via short selling. Importantly, for a taxable investor, relaxed-constraint strategies are even more attractive: Berkin and Luck (2010) and Sialm and Sosner (2018) found that combining relaxation of the long-only constraint with tax-aware rebalancing enhances the tax efficiency of actively managed strategies.

Exhibit 1 compares an actively managed tax-aware relaxed-constraint strategy based on a value–momentum alpha model to three other tax-aware strategies (see Appendix D for further details on simulations of all the strategies). The strategy in the column to the immediate right of the relaxed-constraint is similar to the one originally modeled by Stein and Narasimhan [1999]. The strategy harvests losses while targeting a limited annual tracking error of 1% to the Russell 1000 benchmark. Borrowing from Stein and Narasimhan, hereafter we refer to this strategy as tax-managed passive-indexed (TMPI). The two strategies further to the right are actively managed tax-aware strategies based on the same alpha model as the relaxed-constraint strategy and closely follow the portfolio construction methods of Sialm and Sosner (2018).

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7 We assume that all the trading instruments employed by the strategies are physical (or cash) equities. Although the discussion of taxation of derivatives is outside of the scope of this article, we would like to point out that if derivatives (e.g., swaps or forwards) were used, the tax results would be very different from the ones we describe.

8 See Clarke, de Silva, and Sapra (2004); Jacobs and Levy (2006); Berger (2008); and Ang, Michalka, and Ross (2017).

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Exhibit 1 compares an actively managed tax-aware relaxed-constraint strategy based on a value–momentum alpha model to three other tax-aware strategies (see Appendix D for further details on simulations of all the strategies). The strategy in the column to the immediate right of the relaxed-constraint is similar to the one originally modeled by Stein and Narasimhan [1999]. The strategy harvests losses while targeting a limited annual tracking error of 1% to the Russell 1000 benchmark. Borrowing from Stein and Narasimhan, hereafter we refer to this strategy as tax-managed passive-indexed (TMPI). The two strategies further to the right are actively managed tax-aware strategies based on the same alpha model as the relaxed-constraint strategy and closely follow the portfolio construction methods of Sialm and Sosner (2018).
For calculating tax costs and benefits, we assume that the tax rate applicable to long-term capital gains (losses) and qualified dividend income is 20% and the tax rate applicable to short-term capital gains (losses), interest income, and deductions resulting from in-lieu dividends on short positions is 35%. In our tax-aware portfolio rebalancing, the tax cost function assumes that all the realized gains are taxed immediately at their respective character tax rates and that all the losses immediately offset gains of the same character realized by other unrelated strategies. As a result, short-term capital gains are viewed as more punitive than long-term capital gains, and short-term capital losses are viewed as more attractive than long-term capital losses. In addition, using the methodology from Poterba [1999], we calculate the tax rate applicable to unrealized gains to be approximately 10% (see Appendix A for details). The latter tax rate allows us to estimate the present value of tax costs resulting from future liquidation of unrealized gains. Although we track and report these liquidation tax costs, they are not included in the tax-aware portfolio construction. Similarly, dividend taxes and in-lieu short dividend deductions are not explicitly incorporated into the tax-aware portfolio construction, but they are included in after-tax return calculation. The Taxes panel in Exhibit 1 shows both the level of the active tax (defined as the strategy’s total tax in excess of the tax of the relevant benchmark) and the total tax. Appendix B shows the components comprising the active and total taxes reported in Exhibit 1. Active tax is used in calculation of active after-tax net return. Estimation of active taxes, however, requires estimation of taxes on a benchmark index, which might be a challenging task (for more on this see Sosner, Sullivan, and Urrutia [2018]). Because the purpose of our study is to illustrate the character-deferral decomposition rather than to resolve the complexity of an appropriate benchmark tax calculation, after setting the stage with Exhibit 1, we will focus on total, not benchmark-relative, taxes.

Relative to the Russell 1000 benchmark, the TMPI strategy realizes an annual tax benefit of 30 bps (see the Active Tax line)—approximately half of the 58 bps active tax benefit of the tax-aware relaxed-constraint strategy. This estimate is in line with a number of previous studies simulating a similar TMPI-type strategy. The long-only strategy realizes 26 bps of annual tax liability compared to the relaxed-constraint strategy’s 58 bps of annual tax benefit in excess of the Russell 1000 benchmark. The long–short strategy realizes a very meaningful tax benefit of 2.29% annually relative to its three-month T-bill benchmark. The results for the three actively managed strategies—long-only, relaxed-constraint, and long–short—are qualitatively consistent with findings reported by Salm and Sosner [2018]. Importantly, out of the three beta-one strategies benchmarked to Russell 1000, relaxed-constraint is the most tax efficient.

Because benchmarks themselves tend to generate tax costs, the total tax line in Exhibit 1, which does not subtract the benchmark tax, shows either a higher tax liability or a lower tax benefit. The long–short strategy is the most tax efficient with a tax benefit of 98 bps (see the Total Tax line). Of the three beta-one strategies, relaxed-constraint shows the lowest tax cost at 97 bps, followed by TMPI with a tax cost of 1.25% and long-only with a tax cost of 1.81%.

To be clear, the tax costs of tax-aware strategies need to be compared to tax costs of other investment alternatives. For example, if a taxable investor is looking

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12 See, for example, Stein and Narasimhan (1999); Arnott, Berkin, and Ye (2001a); Berkin and Ye (2003); and Stein, Vadlamundi, and Bouchey (2008). In addition, in our simulation TMPI realizes a small positive alpha, which we find to be fully explained by a momentum exposure resulting from tax awareness. Israel and Moskowitz (2012) and Sialm and Sosner (2018) pointed out that tax-aware optimization tends to introduce a small momentum tilt.

13 Tax costs of the Russell 1000 benchmark result mostly from qualified dividend income and the present value of the future liquidation tax on unrealized gains. To a smaller extent, there are long-term capital gain tax costs associated with periodic reconstitution of the index portfolio. Tax costs of the three-month T-bill benchmark result from interest rate treated as ordinary income.

14 Note that the additional turnover of long-only compared to TMPI causes an additional annual tax cost of 55 bps but also generates 1.8% of the pretax returns, leading to a higher expected after-tax return of the long-only strategy.
for a passive equity allocation, TMPI might be a more attractive alternative than a passive index: Exhibit 1 shows that the tax cost of TMPI is lower and its after-tax return is higher than those of the Russell 1000 benchmark. Similarly, if a taxable investor is looking for an allocation to equities, active or passive, according to our simulations (and subject to the caveats discussed in the last section), tax-aware relaxed-constraint presents a more tax-efficient alternative than either TMPI or tax-aware long-only: Exhibit 1 shows that taxes on relaxed-constraint are lower, and its after-tax returns are higher than those of TMPI and long-only.

Finally, we would like to point out that tax costs and benefits of tax-aware strategies can vary significantly between rising and falling market periods. Exhibit 2 shows how the tax benefits are distributed across up- and down-market years. The relaxed-constraint strategy inherits the features of both the long-only and long-short strategies. Similar to long-only, it realizes significantly higher tax benefits in declining markets than in rising markets. At the same time, because of shorting, it realizes meaningfully lower tax costs than long-only in rising markets when the long-only strategy tends to realize significant tax costs. Similarly, in comparison to TMPI, relaxed-constraint realizes higher tax benefits in down markets and lower tax costs in up markets. Importantly, separating the sample into up- and down-market years does not change our conclusion: Relaxing the long-only constraint results in tax benefits. The relaxed-constraint strategy is more tax efficient than the other two beta-one strategies in both rising and falling markets. Does this relative tax efficiency arise from character or deferral?

**TAX EFFICIENCY OF A TAX-AWARE RELAXED-CONSTRAINT STRATEGY THROUGH THE LENS OF A CHARACTER-DEFERRAL DECOMPOSITION**

**Defining Character and Deferral Benefits**

Exhibit 3 helps to illustrate the concept of current period character and deferral tax benefits. Suppose a portfolio of strategies realizes a total of $100 of capital...
EXHIBIT 3
Illustrative Example of Character and Deferral Amounts

Panel A: Tax-Aware Strategy with a Net Realized Loss

Panel B: Tax-Aware Strategy with a Net Realized Gain
gains, with $50 realized as a long-term gain and $50 as a short-term gain. In Panel A, we add a tax-aware strategy to this portfolio of strategies (to simplify the example, we add the tax-aware strategy rather than allocating to it while at the same time reducing the allocation to other strategies). The tax-aware strategy realizes a long-term gain of $20 and a short-term loss of $30, with a net total capital loss of $10. After combining all the gain realizations, at the overall portfolio level, the amount of long-term realized gains is $70 and the amount of short-term realized gains is $20. Thick borders of the bars in Exhibit 3 mark the long-term and short-term gains before and after adding the tax-aware strategy to the mix. Two things changed because of the addition of the tax-aware strategy. First, the overall level of tax-able realized gains decreased from $100 ($50 long term plus $50 short term) to $90 ($70 long term plus $20 short term). Second, the character of gains shifted favorably from equal amounts of long term and short term ($50 and $50) to mostly long term ($70 versus $20). We define the matching amount of increase in long-term gains and decrease in short-term losses, which in this example equals $20, as the character benefit amount of the tax-aware strategy (shown by the light gray bars). We define the decrease in total realized gains, which in this example is $10, as the deferral benefit amount (shown by the dark gray bar).

It is possible for a tax-aware strategy, rather than allocating a net total loss, to allocate a net total gain. We illustrate such a scenario in Exhibit 3, Panel B. In this scenario, the character benefit amount is $20—this is the matching increase in realized long-term gains and decrease in realized short-term losses at the portfolio level. However, the deferral benefit is negative (i.e., a deferral liability) because the total amount of realized gains at the portfolio level increases from $100 ($50 long term plus $50 short term) to $110 ($80 long term plus $30 short term). In this example, the tax-aware strategy still realizes a character benefit by tilting the balance of the overall portfolio realized gains from short term to long term, but it realizes a deferral liability because the total amount of realized gains increases.

To sum up, the character benefit of a tax-aware strategy results from matching amounts of realized long-term gains and short-term losses, and the deferral benefit (liability) results from a net realized total loss (gain). The algorithm of the decomposition is derived mathematically in Appendix C. The algorithm formalizes and generalizes the ideas illustrated in the previous two examples and captures all possible permutations of gains, losses, income, and deductions.

The product of the character benefit amount and the difference between the short-term and long-term tax rates is defined as the character benefit. The product of the deferral benefit (liability) amount and the applicable tax rate is defined as the deferral benefit. The applicable tax rate is long term if long-term gain exceeds short-term loss or long-term loss exceeds short-term gain; it is short term if short-term gain exceeds long-term loss or short-term loss exceeds long-term gain. If long-term and short-term results are both gains or are both losses, then the appropriate long-term and short-term tax rates are applied to the respective amounts of long-term and short-term gains or losses.

Importantly, in our calculations we make the assumption that when the strategy allocates losses, there are always sufficient gains of the same character from other investments that can be offset by those losses. This assumption helps assess the tax-saving opportunities afforded by the strategy rather than a specific tax situation of any possible investor. These opportunities might materialize differently for different investors, and this is exactly why the character-deferral decomposition is valuable. It is particularly pertinent in the context of tax-aware strategies that seek to deliver tax benefits by realizing short-term capital losses and allocating them to investors. A tax-aware strategy’s character benefit is increased when short-term gains and ordinary income are taxed at a significantly higher rate than long-term gains and qualified dividend income, as in the case of US federal taxes (in particular in the higher tax rate brackets), and when investors have substantial short-term gains from other strategies in their portfolios. The character benefit is reduced when investors have only a small amount of short-term gains or when the difference between short-term and long-term capital gains tax rates is either small (under the alternative minimum tax) or nonexistent (under most state and local taxes).

A deferral benefit can be achieved by a tax-aware strategy that realizes a net capital loss as long as the

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15 Strategies that only realize character benefits might not yield tax benefits for US corporations. The same is true in countries where there is no difference between long-term and short-term capital gains rates (e.g., the United Kingdom, Japan, and many countries in continental Europe).
The Tax Benefits of Relaxing the Long-Only Constraint: Do They Come from Character or Deferral?

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An investor has any type of capital gains from other strategies in the portfolio, long term or short term. This benefit is temporal though, because barring a tax-exempt liquidation of the strategy (e.g., charitable gifting or bequeathal at death), investors will have to pay the deferred taxes upon liquidation. The temporal nature of the deferral benefit is captured by the liquidation tax liability, which increases with the current period’s deferral benefit. Because of the difference between tax rates applicable to current realized gains and the effective tax rate estimated for future realized gains, deferral benefit yields a very real value for a taxable investor.

**Character-Deferral Decomposition of the Tax Benefits of Tax-Aware Strategies**

Exhibit 4 applies the character-deferral decomposition to a representative year for the relaxed-constraint strategy and compares it to the three other strategies. The relaxed-constraint strategy allocates long-term gains and qualified dividend income in the annual amount of 3.5%. We define this amount of excess gains and income as the deferral liability amount. Because this is a liability, in Exhibit 4 we show this amount with a negative sign.

Exhibit 4 shows that in a typical year, relaxed-constraint, and all the other tax-aware strategies, realizes a character benefit. Actively managed strategies tend to realize a larger amount of character benefit than TMPI, and their character benefits increase with leverage: The character benefit amount of relaxed-constraint is an intermediate case between long-only and long–short.

As for the deferral benefit, all the beta-one strategies realize a deferral liability, whereas the long–short strategy realizes a small amount of deferral benefit. The short positions of relaxed-constraint help it achieve a substantially smaller deferral liability amount than long-only. Similar to the character benefit, with respect to the deferral liability, relaxed-constraint achieves an intermediate result between long-only and long–short.

The relaxed-constraint strategy’s character benefit amount is higher than that of long-only and lower than that of long–short. Its deferral liability amount is lower than long-only and higher than long–short.
From these observations it is clear that relaxed-constraint will be more tax efficient than long-only but less tax efficient than long–short. A comparison to TMPI is harder because relaxed-constraint has a much larger character benefit amount than TMPI but also a larger deferral liability amount. To weigh these conflicting results, we must use the respective tax rates applicable to the character and deferral amounts. The next panel in Exhibit 4 shows those rates. The character benefit effective tax rate is the difference between the short-term and long-term capital gains tax rates. The tax rates applicable to the deferral benefit or liability correspond to the category of the excess gain or loss amount—either long term or short term. For example, relaxed-constraint realizes excess long-term gains and qualified dividend income that are taxed at the long-term rate, whereas long–short realizes excess short-term losses and ordinary deductions that are credited with the short-term tax rate.

The total tax benefits and liabilities in Exhibit 4 correspond exactly to those shown in Exhibit 1. However, the character-deferral decomposition in Exhibit 4 allows us to measure the sources of the tax efficiency and tax inefficiency of the four simulated strategies. For example, relaxed-constraint realizes a deferral liability that is 23 bps higher than TMPI—69 versus 46 bps. If investors can use the short-term losses and deductions realized by the strategies efficiently to offset short-term gains and ordinary investment income elsewhere in their portfolios, the character benefit of relaxed-constraint is 62 bps higher than TMPI—85 versus 23 bps. If the investor assigns no value to the character benefit (as, for example, would be the case when the investor has no short-term gains to offset), then the value of the character benefit is zero, and the total preliquidation tax is just the deferral liability.

Note that for the sake of brevity we do not decompose the character and deferral benefits (and liabilities) into those due to capital gains and losses and those due to qualified dividends and ordinary income and deductions. However, this can be easily done using the same logic as we apply here.

Exhibit 5 shows the character-deferral decomposition for an average year during up- and down-market years and during the full strategy simulation.
**Exhibit 5 (continued)**

Character-Deferral Tax Benefit Decomposition in Down- and Up-Market Years, 1988–2017

**Panel B: Comparison of Long-Only to Relaxed-Constraint (RC)**

![Graph showing annualized tax benefit comparison between Long-Only and Relaxed-Constraint in down, up, and all markets over the years 1988–2017.](image)

**Panel C: Comparison of Long-Short to Relaxed-Constraint (RC)**

![Graph showing annualized tax benefit comparison between Long-Short and Relaxed-Constraint in down, up, and all markets over the years 1988–2017.](image)
period.\(^\text{16}\) In Exhibit 2, we saw that relaxed-constraint is more tax-efficient than TMPI in both up and down markets. The character-deferral decomposition in Exhibit 5, Panel A, makes it clear that the source of this relative tax efficiency of relaxed-constraint is the character benefit, which remains high and stable across up- and down-market years—70 and 90 bps, respectively. The TMPI’s character benefit is 40 bps in down markets but is reduced to just 10 bps in up markets, resulting in an average character benefit of 20 bps compared to 80 bps for relaxed-constraint.

Exhibit 5, Panel B, shows why the relaxed-constraint active approach dominates the long-only active approach from the perspective of tax efficiency. First, shorting allows relaxed-constraint to achieve a significantly higher character benefit in both rising and falling markets—especially in up-market years, when relaxed-constraint realizes 70 bps of character benefit compared to only 20 bps of character benefit for long-only. Second, in rising markets, the long-only strategy realizes a deferral liability much larger than that of the relaxed-constraint strategy—1.9% versus 0.9%, respectively.\(^\text{17}\) Although the tax efficiency of relaxed-constraint results in a larger expected liquidation tax cost as compared to long-only, this higher liquidation tax cost is more than compensated by a significantly higher character benefit and a significantly lower deferral liability of relaxed-constraint.

Sialm and Sosner \(^\text{[2018]}\) pointed out that tax benefits of long–short exhibit a positive correlation with market return. Exhibit 5, Panel C, shows that both the character and deferral benefits of long–short are higher in rising than in falling markets. Interestingly, in down markets relaxed-constraint outperforms long–short on all metrics of tax efficiency, whereas in up markets it underperforms. Because the underperformance of relaxed-constraint in up markets is larger than outperformance in down markets and because up markets are more frequent than down markets (25 up years versus only 5 down years during our 1988–2017 sample period), the average tax benefit of long–short is higher. In fact, all categories—character, deferral, and unrealized—contribute to the higher tax benefit of long–short.\(^\text{18}\)

### PRACTICAL SOLUTIONS FACILITATED BY CHARACTER-DEFERRAL DECOMPOSITION

Relaxed-constraint strategies straddle the world of traditional long-only asset management and alternative investing. As a result, these strategies can be thought of as either a replacement of a portion of a traditional equity allocation or an allocation to alternatives. From the pretax perspective, these strategies yield an attractive active return. From the tax perspective, if managed in a separately managed account or in a limited partnership, tax-aware relaxed-constraint strategies can play the role of a tax-efficient core in a core and satellite structure originally proposed by Brunel \(^\text{[2001]}\).\(^\text{19}\) In such a structure, a beta-one tax-efficient core strategy helps offset capital gains realized by tax-inefficient satellite managers. Other authors who advocated the benefits of a core–satellite structure for taxable investors considered TMPI-type strategies as the tax-efficient core strategy.\(^\text{20}\)

Our study makes a number of contributions to this literature on core–satellite portfolio structure. First, we show that barring prolonged market downturns, the tax benefits of both the traditional TMPI strategy and our

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\(^{16}\) The decomposition into character and deferral is nonlinear (the derivation in Appendix C makes this clear). As a result of this nonlinearity, the character-deferral decomposition on average, as we show in Exhibit 5, will be different from the character-deferral decomposition using gains and losses of an average year, as we show in Exhibit 4. However, despite this nonlinearity, the results of character-deferral decomposition are qualitatively and quantitatively similar between the two computation methods.

\(^{17}\) This is consistent with the findings of Sialm and Sosner \(^\text{[2018]}\), who pointed out that in up markets an average short position creates opportunities for a tax-aware strategy to realize losses that can partially offset the gains realized on an average long position.

\(^{18}\) It is important to note that the long–short strategy in our study is simulated at a low average gross notional exposure of about 230% of the net asset value (NAV)—115% long and 115% short. If the gross notional exposure were to increase, all the tax costs and benefits would increase proportionately. For example, if the leverage of our simulated strategy were to double to 230% long and 230% short, in our calculations, the character benefit of long–short would also approximately double. A word of caution is appropriate here: With an increase in leverage, the risks and costs of managing a levered portfolio also increase. Because of these risks and costs, depending on the situation, leveraging up the strategy portfolio beyond a particular level of leverage might become economically untenable.

\(^{19}\) Separately managed account and limited partnership vehicles ensure that losses realized by the strategy can be passed through to the investor. This would not be the case for a regulated investment company, such as a mutual fund or an ETF, which cannot distribute losses to investors.

\(^{20}\) See, for example, Rogers \(^\text{[2001]}\), Stein \(^\text{[2001]}\), Quisenberry \(^\text{[2003]}\), and Bouchey and Pratapnani \(^\text{[2017]}\).
new tax-aware relaxed-constraint strategy come from character and not deferral. Thus, from the tax perspective, if either strategy is used as a beta-one tax-efficient core, the core–satellite structure works best when the satellite managers tend to realize a substantial amount of short-term capital gains. Second, the tax-aware relaxed-constraint strategy realizes a significantly higher character benefit and thus is expected to yield a higher tax benefit than TMPI in the presence of satellite managers with short-term gains. Finally, an actively managed tax-aware long-only strategy is unlikely to function as a beta-one tax-efficient core nearly as effectively as tax-aware relaxed-constraint for two reasons: Its character benefit is a fraction of the character benefits of relaxed-constraint, whereas its negative deferral—acceleration of gains realization—is twice as high as that of relaxed-constraint.21

To sum up, the character-deferral decomposition helps investors and their advisors understand the sources of the tax benefits of a given strategy. Relaxed-constraint, because it realizes substantial and persistent character benefits, offers significant tax benefits to investors with large short-term capital gains from other investments in a core–satellite-like portfolio structure. To such investors, relaxed-constraint is more attractive than TMPI. On the other hand, the decomposition shows that both relaxed-constraint and TMPI partially lose their attractiveness as a tax-management tool at the overall investment portfolio level for investors with long-term capital gains only. This is because in an average year both strategies realize a deferral liability rather than a deferral benefit.22 As a result, our character-deferral decomposition allows for a quick and informative assessment of tax benefits of different tax-aware strategies without modeling various investor-specific situations.

FURTHER CONSIDERATIONS RELATED TO RELAXED-CONSTRAINT STRATEGIES

As always, different approaches have their respective advantages and disadvantages. In our simulations we find that tax-aware relaxed-constraint strategies outperform beta-one long-only strategies as a result of a higher net of costs alpha, lower tax costs of negative deferral, and significantly higher tax character benefits. However, these higher pretax and tax benefits come with a number of caveats.

First, in contrast to TMPI, the relaxed-constraint strategy relies on the performance of a manager’s alpha signals. There is always uncertainty in how well these signals will perform in the future. Therefore, the selection of an active manager might have a sizable impact on after-tax returns. Moreover, even if the active strategy on average continues to perform as expected, it exposes the investor to additional volatility above and beyond the volatility of the passive index. For example, during our sample period value–momentum strategies exhibited strong performance in an average year, yet our simulated relaxed-constraint strategy underperformed the benchmark by close to 9% in 2009 and by approximately 5% in 1991 and 2016. Compared to that, our simulated TMPI strategy underperformed the benchmark by a maximum of 1.7%.

Second, relaxed-constraint strategies might be significantly more costly to manage. In our study we assumed transaction and financing costs on par with those of professional active managers. If investors were to attempt to manage the strategy on their own, their trading and financing costs might be significantly higher than those assumed here.

This brings us to our final point. Managing leveraged strategies in a separately managed account might be costly and inefficient, especially for smaller investors. As a result, it is possible and even likely that such a strategy would be managed by a professional manager in a commingled fund.23 Investment through a commingled fund, however, makes other tax management techniques, such as gifting highly appreciated individual stock positions to charity or strategically realizing unrealized capital gains, virtually impossible. As a result, TMPI or long-only strategies, which are easier to manage in a separate

21 Note that tax-aware rebalancing highly benefits long-only strategies by substantially increasing their after-tax returns. Nonetheless, their after-tax returns and information ratios remain less attractive than those of tax-aware relaxed-constraint strategies, which further benefit from shorting.

22 Stein, Valdamundi, and Bouchey (2008) showed that the tax benefits of TMPI can be enhanced through strategic realization of long-term capital gains. However, those additional benefits again will be character benefits as realized long-term gains are compensated by the ability to realize more short-term losses. It is important to note that in TMPI strategies such long-term capital gains realizations need to be triggered for a tax reason, whereas in actively managed strategies gain realizations are a natural result of the strategies’ turnover.

23 The fund should be organized as a partnership for the investor to benefit from allocation of realized losses.
account, could in practice demonstrate higher tax efficiency compared to what we report here.

**Appendix A**

**TAX RATE APPLICABLE TO UNREALIZED GAINS**

In our derivations we used expected tax rate \( t_E \) to estimate the present value of the future tax liabilities (benefits) resulting from decrease (increase) in the current year’s taxable gains and income. Poterba [1999] derived the following formula for parameter \( t_E \) in the case of a single asset:

\[
    t_E = t_g \frac{p(1-\lambda)(1+r)}{r + p + q - pq}
\]

where \( t_g \) is the statutory tax rate applicable to realized capital gains at a future date when the liquidation occurs, \( p \) is the chance of liquidating the asset in any given year, \( \lambda \) is the probability that the liquidation of the asset does not result in a capital gain tax liability, \( q \) is the probability of step up in the cost basis at death, and \( r \) is the after-tax nominal discount rate.

What is the appropriate level of the tax rate \( t_g \) in this formula? Poterba set \( t_g \) equal to \( t_L \). Under the assumption that future statutory capital gains tax rate remains constant at its current level, setting \( t_g \) equal to \( t_L \) is precisely accurate in the case of liquidation of a single asset (which is exactly the case modeled by Poterba). For portfolio liquidation, there is no guarantee that all the assets in the portfolio will be liquidated at a long-term gain—some of the assets might be held for a period of time shorter than 12 months. Nonetheless, the assumption that for portfolio liquidation the expected level of \( t_g \) is similar to \( t_L \) is plausible under realistic circumstances.

The multiplier of the tax rate \( \frac{p(1-\lambda)(1+r)}{r + p + q - pq} \) can be shown to be between 0 and 1. As the value of this multiplier increases, the future liquidation tax becomes more punitive from the investor’s point of view. An increase in the probability of selling the asset, \( p \), or in the probability of realizing gain when selling the asset, \( 1 - \lambda \), leads to an increase in the rate of gain realization and, assuming a positive after-tax nominal discount rate, \( r \), affects the multiplier positively, thus increasing the expected tax rate. In other words, the higher the chance of realizing the deferred gain, the higher the present value of the tax liability resulting from realization of deferred gains. Increasing the probability of death, \( q \), effectively increases the investment horizon:

There is a correspondence between death and infinite investment horizon—both eliminate the liquidation tax, the former because of the step-up in cost basis, the latter because the liquidation never occurs. Consistent with this, an increase in the probability of death reduces the multiplier and thus reduces the expected tax rate.

In this study, we make the following assumptions:

\[
p = 10\%, \quad 1 - \lambda = 75\%, \quad q = 2\%, \quad r = 3\%, \quad \text{and } t_g = t_L = 20\%\,
\]

resulting in \( t_E = 10.44\% \). We round this rate to 10%. In his numerical example, Poterba [1999, p. 31] also used an effective unrealized gains tax rate of 10%.

**Appendix B**

**Character-Deferral Decomposition of Tax Benefits**

Exhibit B1 shows the components composing active and total taxes. We assume that the tax rate applicable to long-term capital gains (losses) and qualified dividend income is 20% and the tax rate applicable to short-term capital gains (losses), interest income, and deductions resulting from in-lieu dividends on short positions is 35%. We calculate the tax rate applicable to unrealized gains to be approximately 10% (see Appendix A for details of this calculation). Positive values represent tax benefits, whereas negative values represent tax costs.

**Exhibit B1**

Annualized Components of Active and Total Taxes, 1988–2017

<table>
<thead>
<tr>
<th></th>
<th>Relaxed-Constrained</th>
<th>Tax-Managed Passive-Indexed</th>
<th>Long-Only</th>
<th>Long-Short</th>
</tr>
</thead>
<tbody>
<tr>
<td><strong>Active Tax Components</strong></td>
<td></td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>LT Capital Gains</td>
<td>−1.05%</td>
<td>−0.05%</td>
<td>−1.21%</td>
<td>−1.61%</td>
</tr>
<tr>
<td>ST Capital Gains</td>
<td>1.85%</td>
<td>0.49%</td>
<td>0.74%</td>
<td>4.28%</td>
</tr>
<tr>
<td>Qualified Dividend Income</td>
<td>−0.07%</td>
<td>−0.01%</td>
<td>0.00%</td>
<td>−0.39%</td>
</tr>
<tr>
<td>Dividend Expense</td>
<td>0.09%</td>
<td>0.00%</td>
<td>0.00%</td>
<td>0.65%</td>
</tr>
<tr>
<td>Unrealized Gains</td>
<td>−0.25%</td>
<td>−0.13%</td>
<td>0.21%</td>
<td>−0.64%</td>
</tr>
<tr>
<td>Active Tax</td>
<td>0.58%</td>
<td>0.30%</td>
<td>−0.26%</td>
<td>2.29%</td>
</tr>
<tr>
<td><strong>Total Tax Components</strong></td>
<td></td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>LT Capital Gains</td>
<td>−1.27%</td>
<td>−0.27%</td>
<td>−1.44%</td>
<td>−1.61%</td>
</tr>
<tr>
<td>ST Capital Gains</td>
<td>1.88%</td>
<td>0.53%</td>
<td>0.77%</td>
<td>4.28%</td>
</tr>
<tr>
<td>Qualified Dividend Income</td>
<td>−0.54%</td>
<td>−0.49%</td>
<td>−0.47%</td>
<td>−0.39%</td>
</tr>
<tr>
<td>Dividend Expense and Interest</td>
<td>0.09%</td>
<td>0.00%</td>
<td>0.00%</td>
<td>−0.66%</td>
</tr>
<tr>
<td>Unrealized Gains</td>
<td>−1.13%</td>
<td>−1.02%</td>
<td>−0.67%</td>
<td>−0.64%</td>
</tr>
<tr>
<td>Total Tax</td>
<td>−0.97%</td>
<td>−1.25%</td>
<td>−1.81%</td>
<td>0.98%</td>
</tr>
</tbody>
</table>
**APPENDIX C**

**CHARACTER-DEFERRAL DECOMPOSITION OF TAX BENEFITS**

Let \( r_{PT} \) and \( r_{SLT} \) denote the pretax and postliquidation after-tax returns of a strategy, respectively. The strategy realizes net long-term capital gains (losses) in the amount \( g_L \) and net short-term capital gains (losses) in the amount \( g_S \). The strategy also receives qualified dividend income in the amount \( q \) and net ordinary income (loss) in the amount \( i \). Consistent with returns, all the gain, loss, income, and deduction amounts are expressed as a fraction of the strategy’s NAV. Let \( t_L \) denote the lower tax rate applicable to long-term capital gains and qualified dividends and \( t_E \) denote the higher tax rate applicable to short-term capital gains and ordinary income. In addition, let \( t_E \) be the expected tax rate on unrealized gains, as explained in Appendix A.

Using the definition from Poterba [1999], the postliquidation after-tax return of the strategy can be defined as

\[
r_{SLT} = r_{PT} - (g_L + q)t_L - (g_S + i)t_H - (r_{PT} - g_L - q - g_S - i)t_E
\]

where \( r_{PT} - g_L - q - g_S - i \) is the amount of unrealized gain.

The first term on the right-hand side of the equation represents pretax return in a given period, and the remaining three terms represent tax costs (in the case of gains or income) or benefits (in the case of losses or deductions) in the same period. The second term thus measures the tax costs or benefits resulting from the realization of low-taxed characters such as long-term capital gains (losses) and qualified dividend income. The third term measures the tax costs or benefits resulting from the realization of highly taxed characters—short-term capital gains (losses) and ordinary income (deductions). Finally, the last term estimates the tax costs (or benefits) of future liquidation gains (or losses) resulting from the deferral of unrealized capital gains of the current period.

Our main insight is that tax efficiency can be attributed to character and deferral benefits realized in the current period. Character benefits result from realizing losses in highly taxed characters, such as short-term capital losses and ordinary deductions, while at the same time realizing gains and income in low-taxed characters, such as long-term capital gains and qualified dividend income. Deferral benefits arise from postponing gain realization to future periods—such benefits arise from the difference between the reduction in the current tax liability and the increase in the present value of the future tax liability. We recognize that tax efficiency might have an indirect adverse effect on the level of pretax returns of a strategy. However, such an effect is rarely measureable under realistic circumstances. As a result, our proposed character-deferral decomposition focuses on observable tax outcomes resulting from trading or holding of the strategy positions and ignores the potential effects on the pretax return.

There are three scenarios of interest for the purpose of character-deferral decomposition:

1. \( g_L + q \) and \( g_S + i \) do not have a different sign (this includes 0 for either one or both sums)

   In this case, all the tax benefits (liabilities) come from deferral (acceleration) of gains:

   \[
   \frac{(-1)[(g_L + q)t_L + (g_S + i)t_H]}{character benefit (liability)}
   \]

2. \( g_L + q \) and \( g_S + i \) have different sign and \( |g_L + q| \geq |g_S + i| \)

   In this case, the character benefit is applied to the amount of long-term gains and qualified dividends, and the excess short-term capital and ordinary losses yield the deferral benefit:

   \[
   \frac{(-1)[(g_L + q)t_L + (g_S + i)t_H]}{character benefit (liability)} = \frac{(g_L + q)(t_L - t_E) + (1)(g_L + q + g_S + i)t_H}{deferral benefit (liability)}
   \]

   For example, if short-term capital losses and ordinary deductions add up to 15% of the NAV and long-term capital gains and qualified dividends add up to 10% of the NAV, the decomposition will identify 10% multiplied by the difference between high and low tax rates as the character benefit and 5% of excess short-term gains and deductions multiplied by the high tax rate as the deferral benefit. This deferral benefit is temporal, as reflected by the increase in the expected tax costs of future liquidation measured separately by \(-t_E - g_L - q - g_S - i\).

3. \( g_L + q \) and \( g_S + i \) have different sign and \( |g_L + q| > |g_S + i| \)

   In this case, the character benefit is applied to the short-term capital and ordinary losses and the excess long-term gain and qualified dividends result in a negative deferral benefit, or acceleration of tax liabilities to the current period:

   \[
   \frac{(-1)[(g_L + q)t_L + (g_S + i)t_H]}{character benefit (liability)} = \frac{(-1)[(g_L + q)(t_L - t_E) + (1)(g_L + q + g_S + i)t_H]}{deferral benefit (liability)}
   \]

For example, if short-term capital losses and ordinary deductions add up to 10% of the NAV and long-term capital gains and qualified dividends add up to 15% of the NAV, the decomposition will identify 10% multiplied by the difference between high and low tax rates as the character benefit and 5% of excess long-term
gains and qualified dividends multiplied by the low
tax rate as the deferral liability (i.e., acceleration of net
gains and income to the current period). Similar to the
mentioned deferral benefit, this deferral liability is
temporal, which is now reflected by the decrease in
the expected tax costs of future liquidation.

These three cases can be conveniently summarized by
a single formula:

\[ \theta = \min \left( \max \left( \frac{g_t + i}{g_t + g_t - g_t + i}, 0 \right), 1 \right) \]

Scenarios 1, 2, and 3, presented earlier, correspond to
\( \theta = \frac{g_t + i}{g_t + g_t - g_t + i} \), \( \theta = 1 \), and \( \theta = 0 \), respectively. For compu-
tational purposes, this formula might serve as a convenient
replacement of the conditional statements in 1, 2, and 3.

**APPENDIX D**

**EMPIRICAL METHODOLOGY**

Our methodology closely follows Sialm and Sosner
(2018) with one addition: We also considered a strategy that
is passive with respect to alpha but is actively tax managed.
This section describes the methodology we used to construct
investment strategies.

**Active Strategies**

Stein and Narasimhan (1999) made a distinction between
active alpha and active tax management. According to Stein and
Narasimhan, a manager who is active with respect to security
selection but ignores the tax consequences of trading is "active
with respect to tax." Active tax management seeks to improve
after-tax returns via acceleration of capital losses and deferral
of capital gains, a technique otherwise known as loss-harvesting.
A manager who is passive with respect to security selection
(e.g., seeking only to match an index) might thus still be active
with respect to tax. Borrowing from Stein and Narasimhan,
we call such strategies TMPI. In this study, we modeled a
TMPI strategy along with actively managed tax-aware value-
momentum strategies considered by Sialm and Sosner.

**Value and Momentum Strategies**

We focused on quantitative strategies that combine
value and momentum style factors. Value investing has been
researched extensively in academia (see, among many others,
Fama and French 1992) and has been widely applied in the
asset management industry. We used the most frequently
used academic measure of equity value, the book-to-market
ratio. Consistent with Asness and Frazzini [2013], we scaled
the book value of a company by its most recent market
capitalization.

Momentum effects in US equities were first docu-
Specific definitions of relative strength in past performance
may vary; in our study, we used the simplest and the best-
known measure of momentum: the total return over the
preceding 12 months, excluding the most recent month.
Combining value and momentum strategies is particu-
larly beneficial because these strategies tend to exhibit nega-
tive correlation. In our alpha model, we combined value and
momentum with equal risk weights. This approach is justified
because, under realistic portfolio constraints and transaction
costs, integrated strategies dominate naïve combinations of
individual factor tilts before taxes (see Clarke, de Silva, and

From an economic perspective, these styles have per-
sisted across different asset classes, markets, and time periods
(see Asness, Moskowitz, and Pedersen [2013] and Asness et al.
[2015]). From a tax perspective, value signals are negatively
related to past returns and momentum signals are positively
related. As a result, in addition to their established pretax eco-
nomics, these factors exhibit interesting tax dynamics (Israel
and Moskowitz [2012]).

**Alpha Model**

In our study, we begin the portfolio construction pro-
cess with an alpha model that yields stock-level alphas. Black
and Litterman (1992) showed that alpha forecasts consistent
with risk and correlation forecasts are more effective in port-
folio optimization. Jones, Lim, and Zangari (2007) adapted
this insight to factor investing in the context of equity port-
folios. We used their methodology to obtain stock-level alphas
because our tax-aware approach relies on optimization to
achieve a balance between the pretax alpha and the tax costs
of portfolio rebalancing.

The starting point for our alpha forecasts is a model
portfolio, \( \hat{\nu} \), which is long attractive stocks and short
unattractive stocks. The determination of relative attractiveness for each factor in the model—in our case, value and momentum factors—is done at the end of every calendar month using data available at that time. The model portfolio \( \tilde{\mathbf{v}} \) is a vector of long–short portfolio weights that captures the combined exposures of stocks to value and momentum factors. For example, stocks with high value and momentum exposures exhibit positive weights, whereas stocks with low value and momentum exposures exhibit negative weights.

Following the methodology of Jones, Lim, and Zangari (2007), at every month-end, we converted the model portfolio \( \tilde{\mathbf{v}} \) into a vector of stock-level alphas by multiplying it by the stock-level covariance matrix \( \Sigma \) (from MSCI Barra’s USE3L risk model discussed in more detail later in the Appendix):

\[
\alpha = \Sigma \tilde{\mathbf{v}}
\]

The alpha forecasts used in the optimization rely on only the information available at the time of portfolio construction.

Model Portfolio Construction

We construct the factor-based model portfolio \( \tilde{\mathbf{v}} \) as follows. We first construct long–short value and momentum factor portfolios within the US large-cap stock universe that approximately corresponds to Russell 1000 Index constituents.\(^{24}\)

For our measure of value, we use the book-to-market ratio, in which the denominator is the most recently available market capitalization (Asness and Frazzini 2013). Book values are from the Compustat annual files. We lag the annual book values by six months relative to the fiscal year-end to ensure that they are available at the time of model construction. For our measure of momentum, we use the total return over the last 12 months, excluding the most recent month.

The book-to-market and momentum raw scores are then turned into industry-relative ranks. For the industry classification, we use the industry levels of the Global Industry Classification Standard (GICS), developed by MSCI and S&P. The GICS classification has four levels—sector, industry group, industry, and subindustry—with sector the most aggregated and subindustry the most granular. Historically, the number of GICS industries ranged from 59 before 1999 to 70 in 2017. We rank the stocks within each industry according to their book-to-market and momentum scores. These ranks within each industry are de-meaned (by subtracting the average rank within an industry) and standardized (by dividing by the standard deviation of the ranks within an industry) to create an industry-neutral portfolio. Because the weights of a portfolio sum to zero within each industry, the weights of the whole portfolio also sum to zero. These long–short portfolio weights for the two signals are denoted by \( v_{VAL} \) for value and \( v_{MOM} \) for momentum. This rank-based portfolio formation method is similar to that of Asness et al. (2015).

We use this method of portfolio construction for several reasons. First, this approach is consistent with that of Asness et al. (2015), who considered within-industry stock selection and industry selection as distinct asset classes. Second, by using ranks, we define a view on every stock in the universe, which helps the tax-aware optimization make trade-offs between alpha forecasts and tax costs. A decile or a quintile approach would leave many stocks without alpha views, which would make the alpha-versus-tax trade-off less precise.

The value and momentum portfolios are then normalized by their respective volatility forecasts:

\[
\tilde{v}_{VAL} = \frac{1}{\sigma_{VAL}} v_{VAL}, \quad \tilde{v}_{MOM} = \frac{1}{\sigma_{MOM}} v_{MOM}
\]

Using a covariance matrix produced by the MSCI Barra USE3L risk model, we compute the volatility forecasts. The risk model yields a stock-level covariance matrix every month. We use this covariance matrix, \( \Sigma \), lagged by one month, to compute \( \sigma_{VAL} \) and \( \sigma_{MOM} \) as follows:

\[
\sigma_{VAL} = \sqrt{\tilde{v}_{VAL}^T \Sigma \tilde{v}_{VAL}}, \quad \sigma_{MOM} = \sqrt{\tilde{v}_{MOM}^T \Sigma \tilde{v}_{MOM}}
\]

Lagging the covariance matrix by one month accounts for the fact that it takes a few days after the month-end for the risk model to be released, and it ensures that the covariance matrix used in volatility calculations is available at the time of factor portfolio formation.

The model portfolio is then constructed as an equal-weighted average of the value and momentum portfolios:

\[
v = \frac{1}{2} \tilde{v}_{VAL} + \frac{1}{2} \tilde{v}_{MOM}
\]

Finally, the model is normalized by its predicted volatility:

\[
\tilde{v} = \frac{1}{\sigma} v
\]
The model volatility \( \sigma \) is computed in the same way as the value and momentum factor volatilities.

**Covariance Matrix**

The estimation of stock-level covariance matrixes is intrinsically challenging. The number of stocks is large compared with the number of periods available for estimating the covariance matrix. Moreover, the return data are sparse because different stocks have return series of different lengths. These problems become even more pronounced when stock covariance matrixes are used in portfolio optimization, which effectively relies on the inverse of the covariance matrix. Multifactor models represent stock correlations using a small number of factors and thus significantly reduce the number of estimated parameters. Fan, Fan, and Lv (2008) showed that applying a factor structure to covariance matrix estimation is particularly advantageous when the covariance matrix is used for portfolio optimization.

Because the estimation of stock covariance matrixes was not the focus of our study, similar to Asness et al. (2015), we used covariance matrices from MSCI Barra, which applies a multifactor approach to covariance matrix estimation. The MSCI Barra USE3L risk model provides a covariance matrix of all stocks traded on US exchanges. The model uses 52 industries and 13 risk factors—including volatility, size, value, momentum, and leverage—to capture the common variation in stock returns. The model is updated monthly using information about stock returns and fundamentals available at month-end. As indicated by the release date of the model handbook (Barra 1998), the model’s factor structure was chosen before February 1998.

Similar to the Fama–MacBeth (1973) procedure, the model first computes factor loadings using past data and then estimates cross-sectional regressions of stock-level returns on those factor loadings. The regression coefficients estimated in each period are factor returns for that period, and the regression residuals are stock-specific returns for that period. Time-series factor returns up to that period are then used to compute a forward-looking forecast of the factor covariance matrix. Stock-specific returns up to that period are used to compute forward-looking stock-specific volatility forecasts. More details about the model estimation are available in the model handbook (Barra 1998).

**Portfolio Construction**

Using stock-level alpha and covariance matrix estimates, we constructed TMPI, long-only, relaxed-constraint, and long–short portfolios, updating them every month-end. The portfolio weights of the individual securities in TMPI and long-only strategies are all positive and sum to 100%. The weights of the securities in a relaxed-constraint portfolio can be negative, but the aggregate weight of the long positions and the aggregate weight of the short positions cannot exceed certain levels. For example, a 130/30 relaxed-constraint portfolio holds long positions amounting to 130% and short positions amounting to 30% of the portfolio’s net asset value. The main difference between long-only and 130/30 relaxed-constraint portfolios is that in a long-only portfolio, the sum of absolute weights is 1.0, whereas in a 130/30 portfolio, it is 1.6 (130% long plus 30% short). For the TMPI, long-only, and relaxed-constraint portfolios, the betas relative to the Russell 1000 Index are constrained to be close to 1.0. The weights of a long–short portfolio sum to zero. The betas of the long–short portfolios relative to the Russell 1000 are constrained to be close to zero. There is no constraint on the gross notional value (i.e., the sum of the absolute values of all the long and short weights) of the long–short portfolio. Thus, the long–short portfolio’s gross notional value adjusts to a particular level of target risk.

Note that optimization allows us to construct both tax-agnostic and tax-aware strategies. Tax awareness is implemented through a penalty term that incorporates tax costs into the portfolio’s objective function, assuming a tax-aversion coefficient of 0.5 for TMPI and 0.1 for actively managed long-only, relaxed-constraint, and long–short strategies. For active strategies, we lower the tax-aversion coefficient to maintain a balance with pretax alpha.

For TMPI strategies, the optimization problem is defined as follows:

\[
\max_{w_i} -\gamma T - \epsilon
\]

subject to:

\[
\sum_{i} w_i \sigma_{ij} \leq TE^2
\]

\[
\sum_{i} (b_i + w_i) = 1
\]

\[
0.98 \leq \sum_{i} (b_i + w_i) \beta_i \leq 1.02
\]

where \( w_i \) corresponds to the active portfolio weight of security \( i \), \( \gamma = 0.5 \) is the tax-aversion coefficient, \( T \) is the tax cost of rebalancing the portfolio in the current period, \( \epsilon \) is transaction costs described in detail later, \( \sigma_{ij} \) is the covariance between the returns of securities \( i \) and \( j \) derived from MSCI Barra’s risk model, \( TE \) is the target tracking error of 1% annually, \( b_i \) is the benchmark weight of security \( i \), and \( \beta_i \) corresponds to the beta of security \( i \) with respect to the Russell 1000 Index predicted by MSCI Barra risk model. The first term in the objective function effectively rewards the realization of losses and penalizes the realization of gains. Short-term losses are rewarded more than long-term losses, and short-term gains...
are penalized more than long-term gains. In addition, the higher the tax-aversion coefficient, the greater the importance of reducing tax costs (or creating tax benefits) as compared to transaction costs.

Note that both the covariance and the beta estimates are point-in-time forward-looking estimates. In addition, we lagged these estimates by one month to ensure that the risk model data were released before the portfolio construction date.

The optimization problem for long-only and relaxed-constraint strategies is defined as follows:

$$\max_{w_i, w_j} \sum_i w_i \alpha_i - \gamma T - \epsilon$$

s.t.

$$\sum_i \sum_j w_i w_j \sigma_{ij} \leq T E_i$$

$$\sum_i (w_i + w_j) = 1$$

$$\sum_i \sum_j \sigma_{ij} = 1 + 2L$$

$$0.98 \leq \sum_i \sum_j \sigma_{ij} \beta_i \leq 1.02$$

where $w_i$ and $\alpha_i$ correspond to the active portfolio weight and the alpha of security $i$, $\gamma$ is the tax-aversion coefficient (equal to 0 for tax-agnostic optimization and 0.1 for tax-aware optimization), $T$ is the tax cost of rebalancing the portfolio in the current period, $\epsilon$ is transaction costs, $\sigma_{ij}$ is the covariance between the returns of securities $i$ and $j$ derived from MSCI Barra’s risk model, $T E_i$ is the target tracking error of 4% annually, $\beta_i$ is the benchmark weight of security $i$, $L$ is the constraint on short selling (e.g., $L = 0$ for a long-only portfolio and $L = 0.3$ for a 130/30 relaxed-constraint portfolio), and $\beta_i$ is the beta of security $i$ with respect to the Russell 1000 Index predicted by MSCI Barra risk model.

For the long–short strategy the weights are not defined relative to a benchmark, and thus the optimization problem is defined slightly differently:

$$\max_{w_i, w_j} \sum_i w_i \alpha_i - \gamma T - \epsilon$$

s.t.

$$\sum_i \sum_j w_i w_j \sigma_{ij} \leq R^2$$

$$\sum_i w_i = 0$$

$$-0.02 \leq \sum_i \sum_j \sigma_{ij} \beta_i \leq 0.02$$

where $R$ is the target risk level of 4% annually for the long–short portfolio.

The tax cost of rebalancing a portfolio is defined as follows:

$$T = t_{LT} g_{LT} + t_{ST} g_{ST}$$

where $t_{LT}$, $t_{ST}$ are the long- and short-term capital gain tax rates, respectively, and $g_{LT}$, $g_{ST}$ are the net long-term and short-term capital gains computed from individual tax lots, respectively. The tax burden, $T$, is expressed as a proportion of the strategy’s net asset value. Although dividend taxes and in-lieu short dividend deductions are not explicitly incorporated into the optimization, they are included in the reported after-tax returns.25 Unrealized gains are also not included in the optimization problem. This implies that at the portfolio construction stage we assume that the tax rate applicable to unrealized gains is zero. In the article, we apply a 10% tax rate to unrealized gains for the purpose of after-tax return calculation. This tax rate estimate, however, is highly dependent on numerous assumptions outlined in the main article, which is why we leave it out of portfolio construction decisions.

Several studies have documented that the choice of accounting method for tax lot selection has a nontrivial effect on after-tax returns (Dickson, Shoven, and Sialm 2000; Berkin and Ye 2003; Israel and Moskowitz 2012). Because the effects of tax lot accounting are not central to our conclusions and have been analyzed elsewhere, we use the highest in, first out tax lot accounting method throughout this article.

**Tax Rate Assumptions**

The tax rates on short-term and long-term capital gains are assumed to be 35% and 20%, respectively.26 All dividends paid on long positions are assumed to be qualified dividend income; dividends paid on short positions are not, so the tax rates are 20% and 37%, respectively. The capital gains tax rate for long-term capital gains is 0% and 20%, respectively. Short-term capital gains are taxed at the same rate as ordinary income. Note that many states impose additional taxes on capital gains, which are not included in these rates.

25 In analysis not reported here, for the type strategies modeled in this study, we could not find convincing evidence that penalizing dividend exposure is beneficial for after-tax strategy returns.

26 IRC §§ 1222 and 1223 define the holding periods for the determination of long-term and short-term capital gains and losses, and IRC § 1 provides the applicable tax rates for short-term and long-term gains. As of this writing in 2018, under IRC § 1, the top-bracket tax rates for long-term and short-term capital gains are 20% and 37%, respectively. In addition to this base rate, under IRC § 1411, a 3.8% Medicare surtax is imposed on net investment income for modified adjusted gross income levels above $200,000 for individuals, $250,000 for couples filing jointly, and $125,000 for spouses filing separately. Note that many states impose additional taxes on capital gains, which are not included in these rates.
income and are thus taxed at a 20% rate. This assumption is consistent with strategies that use relatively long holding periods, as does the combined value and momentum strategy described earlier. Gains on short positions are taxed as short-term capital gains, regardless of the holding period for short positions. In-lieu dividends paid on short positions are treated as an interest expense offsetting ordinary investment income. Because the portfolios are rebalanced monthly, we assume that the trades are not subject to the wash-sale rule, which defers capital losses for tax purposes if the investor reestablishes a position disposed of at a loss within a period beginning 30 days before and ending 30 days after the date of the disposition, excluding the day of disposition.

In the base case, we assume that realized losses can be used immediately to offset capital gains of the same type (i.e., short-term losses with short-term gains and long-term losses with long-term gains) elsewhere in the investor’s portfolio. This assumption means that an investor who realizes a $10 short-term (long-term) capital loss can achieve a tax benefit of $3.50 ($2.00) in the current year. Thus, these results are relevant for investors who realize sufficient short-term and long-term capital gains from other investment sources. The benefits of tax losses are smaller if the investment vehicle is structured as a mutual fund or if the investor does not have any other capital gains in the portfolio. In such cases, the remaining capital losses must be carried forward to future years. We discuss these cases in the section on robustness tests.

### Management Fee, Transaction Cost, and Leverage Cost Assumptions

All the results in the article are reported gross of management fees. We use a simple transaction costs model informed by the academic research, such as that by Almgren et al. (2005). Transaction costs per dollar traded in basis points are modeled as

$$\text{transaction costs}_{i,t} = 15 + 0.075 \times \text{VIX}_{t} + 2.5 \times \text{risk}_{i,t} \times \sqrt{\frac{T\$_{i,t}}{\text{DTV}_{i,t}}}$$

where $\text{VIX}_{t}$ is the most recent VIX level known on the date of the trade, $\text{risk}_{i,t}$ is the specific volatility of stock $i$ as estimated by MSCI Barra USE3L model, and $T\$_{i,t}$ and $\text{DTV}_{i,t}$ are the dollar trade size and dollar daily trading volume of stock $i$, respectively.

We use the results of Frazzini, Israel, and Moskowitz (2015) to confirm our model assumptions. They estimated that the average market impact cost for a large institutional investor following quantitative strategies in the large-capitalization developed-markets universe was less than 20 bps of the trading value over the period from 1998 to 2013. These market impact costs correspond to average trade sizes of around half a million and amounting to around 1% of the average daily trading volume. If we substitute 20 for VIX—the average VIX level in the 30 years from 1986 to 2015; 20 for specific risk—the average MSCI Barra specific risk of large-capitalization stocks in percentage points; and 1% for the trade as a fraction of DTV, we obtain a 21.5 bp transaction cost.
cost, on average. For a few trades that represent a high fraction of DTV (e.g., 5%), the cost becomes 27.7 bps.

For the costs of leverage, following Sorensen, Hua, and Qian (2007) and Sialm and Sosner (2018), we use the assumption of 100 bps financing cost per unit of one-sided leverage. For a 130/30 relaxed-constraint portfolio, this implies an annual cost of 30 bps cost (i.e., 0.3 times 100 bps). Similarly, for a long–short portfolio with a gross notional value of 200%, the financing cost is estimated at 100 bps (i.e., 2.0 gross notional value divided by 2, to obtain leverage, times 100 bps).

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