Issues: Pricing and Aftermarket Trading Considerations" (Working paper, Southern Methodist University, 1988).

10. The average discount from NAV for existing funds (S&P Closed-End Equity Fund Index) was calculated on each new fund's initial offering date and the mean discount for these existing funds across all these days was then computed.

Appendix Closed-End Equity Fund Initial Public Offerings by Type of Fund, Issue Size and Exchange Listing, January 1986–June 1987

Name of Fund	*Classification*[a]	*Issue Size*[b]	*Exchange Listing*
Asia Pacific Fund	International	$ 86.5	NYSE
Blue Chip Value Fund	Diversified	85.0	NYSE
Clemente Global Growth Fund	Diversified	60.0	NYSE
Counselors Tandem Securities Fund	Specialized	44.0	NYSE
Cypress Fund	Specialized	85.0	NYSE
Duff & Phelps Selected Utilities	Specialized	1,200.0	NYSE
Ellsworth Convertible Growth and Income Fund	Specialized	45.0	AMEX
EquityGuard Stock Fund	Diversified	18.5	AMEX
First Financial Fund	Specialized	92.0	NYSE
France Fund	International	90.0	NYSE
Gabelli Equity Trust	Specialized	400.0	NYSE
Germany Fund	International	75.0	NYSE
Global Growth & Income-Capital	Diversified	50.0	NYSE
Growth Stock Outlook Trust	Diversified	125.0	NYSE
H&Q Healthcare Investors	Specialized	55.0	NYSE
Hopper Soliday Fund[c]	Diversified	40.0	NYSE
Italy Fund	International	66.0	NYSE
Liberty All-Star Equity Fund	Diversified	510.0	NYSE
Lincoln National Convertible Securities	Specialized	90.0	NYSE
Malaysia Fund	International	84.0	NYSE
Morgan Grenfeld SMALLcap Fund	Specialized	50.0	NYSE
Nicholas-Applegate Growth Fund	Diversified	100.0	NYSE
Pilgrim Regional BancShares	Specialized	90.0	NYSE
Quest for Value Fund-Capital	Diversified	225.0	NYSE
Regional Financial Shares Fund	Specialized	100.0	NYSE
Royce Value Trust	Diversified	100.0	NYSE
Scandinavia Fund	International	65.0	AMEX
Schafer Value Trust	Diversified	110.0	NYSE
Scudder New Asia Fund	International	84.0	NYSE
TCW Convertible Securities Fund	Specialized	200.0	NYSE
Taiwan Fund	International	24.4	AMEX
Templeton Emerging Markets Fund	International	100.0	AMEX
Worldwide Value Fund	Diversified	60.0	NYSE
Zweig Fund	Diversified	300.0	NYSE

a. Fund classifications are those used by the *Wall Street Journal* in the weekly section "Publicly Traded Funds."
b. Millions of dollars (prior to the exercise of any over-allotment option).
c. Originally Decision Capital Fund.

On the Distribution of Financial Futures Price Changes

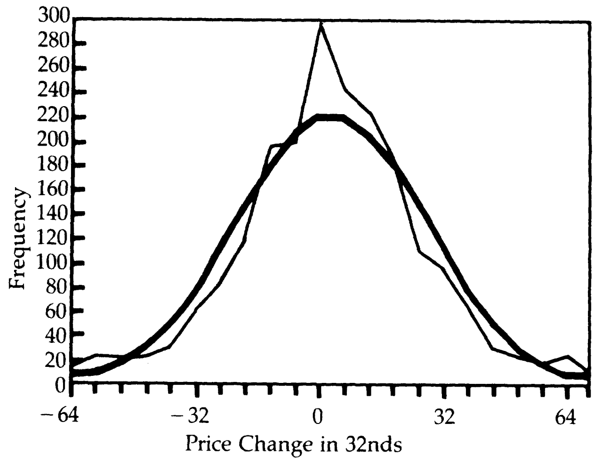
by Andrew J. Sterge, CoreStates Financial Corporation, Philadelphia

Among the victims of the October 1987 market crash were the popular and convenient assumptions of nearly continuous and normally distributed price change processes. Because these assumptions underlie many sophisticated risk assessment, option pricing and trading models, these models should be reconsidered in the light of more realistic characterizations of price change distributions.

This note investigates the behavior of Treasury bond, 10-year Treasury note and Eurodollar futures price changes over four different time intervals, ranging from overnight to one month.[1] Simple inspection reveals that these distributions exhibit both greater central tendency and greater extreme behavior than normal distributions. Statistical goodness-of-fit tests confirm these observations.

1. Footnotes appear at end of article.

Figure A Bond Futures versus Fitted Normal Price Changes (one-day close-to-close)



Data

I examined price changes over four intervals—close-to-open, one-day close-to-close, five-trading-day close-to-close and 22-trading-day close-to-close. The last two intervals represent one-week and one-month price changes, respectively. For these two intervals, I compiled all successive observations, rather than nonoverlapping observations, in order to enrich the distributions; any bias between Monday-to-Monday and Tuesday-to-Tuesday closing prices, for example, is ignored.[2]

I obtained time series for perpetual futures prices from Commodity Systems, Inc.[3] For the bond contract, I used prices from September 4, 1979 to January 26, 1987; that starting date was chosen because the third quarter of 1979 was the beginning of a marked increase in interest rate volatility. The Eurodollar price data start from the contract's inception—December 9, 1981—and end January 26, 1988. The 10-year note data also start from the contract's inception—May 3, 1982—and end August 18, 1988. The bond data comprise 2,120 trading days, the note data 1,593 days and the Eurodollar data 1,535 days.

I excluded night trading from the bond contract data to make overnight statistics more meaningful; statistics from the other time intervals were unaffected by this exclusion. Day trading statistics for the note contract were unavailable, however; for "overnight" price changes after May 1, 1987, I used the change from the 3:00 p.m. close to the night opening.

Properties of Price Change Distributions

Figures A, B and C depict the differences between fitted normal and one-day close-to-close price change distributions for the Treasury bond, Treasury note and Eurodollar perpetual futures contracts. Differences between fitted normal and the three other time interval price change distributions look similar.

Note that there are consistently more very small and very large absolute price changes, but fewer intermediate-sized price changes, than predicted by normality. Statistically, the hypotheses that these price changes are normally distributed are soundly rejected for all four time intervals and for all three futures contracts (see Table I). The consistency with which the empirical distributions differ from normality shows that non-normal behavior is not limited to short-term price movements but exists for long-term movements as well.

Tables II, III and IV present the price change distributions in percentile format. Models incorporating these empirical distributions will be more realistic than those using hypothetical normal price change distributions. Particularly affected will be those models relying on volatility estimates (option valuation and risk assessment immediately come to mind). Indeed, the sample standard deviation of a fat-tailed distribution—that is, a distribution with infinite variance—is an estimate of something nonexistent. Sample standard deviations of these distributions are therefore irrelevant and meaningless calculations. More appropriate dispersion measures for random variables with fat-tailed distributions are mean absolute deviations and percentile ranges.

Note, however, that disclaiming the validity of sample standard deviation depends on actually proving the distributions involved have infinite variances. This has only been assumed here; further information is provided by research on stable paretian distributions and their appropriateness to stock and commodity price changes.[4]

Figure B Note Futures versus Fitted Normal Price Changes (one-day close-to-close)

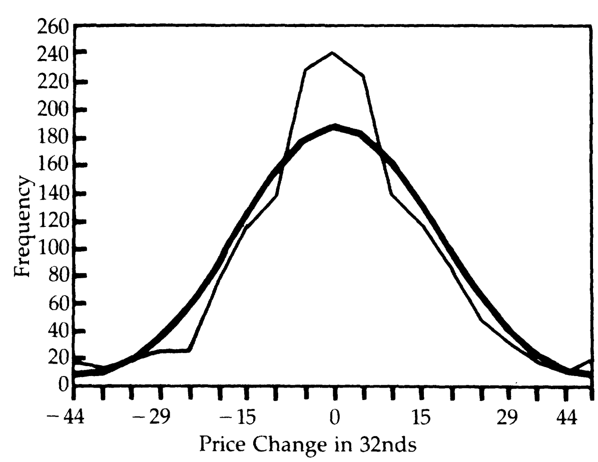
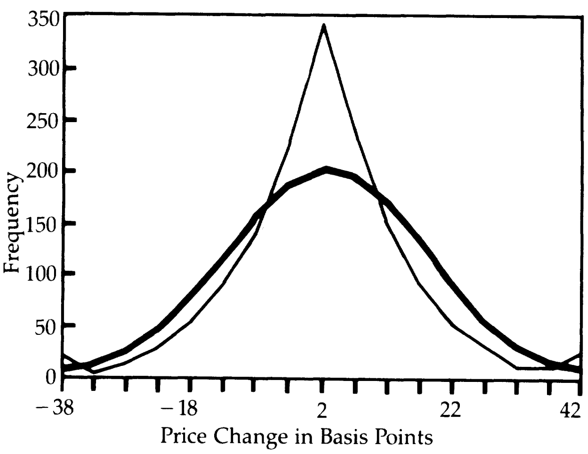


Figure C Eurodollar Futures versus Fitted Normal Price Changes (one-day close-to-close)



Extreme Behavior

The October 1987 cataclysm necessitates an appreciation for the likelihood and constitution of quick and extreme moves in the market. This is especially important given the evidence of non-normality in price changes and the presence of fat tails in particular. Indeed, analysis of the data shows that the bond contract compiled 56 observations of more than 36 32nds, up or down, from close-to-open. In contrast, only 17 observations of greater magnitude than 36 32nds were expected from the fitted normal. For close-to-close price changes, there were 76 observations greater than 58 32nds, whereas only 34 such observations were expected from the fitted normal.

For the note contract, there were 41 overnight observations over 28 32nds, versus the 14 expected, and 37 close-to-close observations over 42 32nds, versus the 14 expected. Lastly, for the Eurodollar, there were 26 overnight observations over 30 basis points, versus 12 expected, and 47 close-to-close observations over 38 basis points, versus 19 expected. In all cases, more than twice as many extreme price changes occurred as were expected under the assumption of normality.

Table I Chi-Squared Statistics*

	Bond		*Note*		*Eurodollar*	
	X^2	*d.f.*	X^2	*d.f.*	X^2	*d.f.*
Close-to-Open	300.0	16	181.2	10	491.3	11
Close-to-Close	134.8	19	98.0	16	238.3	14
Weekly	83.9	26	57.9	12	88.2	34
Monthly	128.4	81	37.4	22	45.2	45

* With one exception, the null hypothesis is rejected at the 0.001 or lower significance level; the hypothesis for monthly note data is rejected at the 0.025 level.

Table II Treasury Bond Percentiles (in 32nds)

Percentile	*Close-to-Open*	*Close-to-Close*	*Weekly*	*Monthly*
1	−43	−63	−82	−294
5	−22	−40	−57	−189
10	−16	−29	−43	−145
20	−9	−18	−25	−96
30	−6	−11	−16	−60
40	−3	−5	−8	−30
50	0	0	0	−1
60	3	5	7	27
70	6	11	16	56
80	9	18	26	90
90	15	30	41	147
95	23	39	56	199
99	38	64	90	316

Straightforward compilations such as the preceding serve to illustrate the inappropriateness of using normal distribution in models of the financial markets. The new disciplines of chaos and catastrophe theory may offer some significant insights in this regard.[5] Especially promising are suggestions of alternatives to the random walk hypothesis, ones in which extreme events are naturally occurring, not aberrant, phenomena. Even though no such alternative is close to being fully or formally developed, the potential these theories show for further insight into price behavior, extreme discontinuities and all, is encouraging.

Conclusion

Graphic evidence and statistical tests suggest that Treasury bond, 10-year Treasury note and Eurodollar futures price changes are non-normally distributed, exhibiting greater central tendency as well as greater extreme behavior than expected from normal distributions. Moreover, both long and short-term fluctuations in bond, note and Eurodollar futures prices are non-normal.

The empirical price change distributions have fat

Table III 10-Year Treasury Note Percentiles (in 32nds)

Percentile	*Close-to-Open*	*Close-to-Close*	*Weekly*	*Monthly*
1	−34	−42	−89	−192
5	−16	−26	−54	−122
10	−11	−19	−43	−91
20	−6	−12	−28	−59
30	−4	−6	−17	−35
40	−2	−3	−8	−14
50	0	0	1	9
60	2	4	11	27
70	4	7	19	49
80	7	12	29	73
90	10	19	45	114
95	15	26	61	148
99	27	46	103	224

Table IV Eurodollar Percentiles (in basis points)

Percentile	*Close-to-Open*	*Close-to-Close*	*Weekly*	*Monthly*
1	−23	−41	−78	−165
5	−11	−22	−52	−112
10	−8	−16	−38	−82
20	−5	−9	−23	−46
30	−3	−5	−15	−26
40	−1	−2	−6	−6
50	0	0	1	9
60	2	2	9	24
70	4	5	16	44
80	6	9	26	67
90	10	16	41	101
95	15	23	57	133
99	30	44	103	200

tails, quite unlike the vanishing tails of normal distributions. Therefore the frequency of extreme observations cannot be ignored in characterizing the expected behavior of futures prices. Evidence suggests that very large (three or more standard deviations from the norm) price changes can be expected to occur two to three times as often as predicted by normality.

Finally, the abundance of "catastrophic" events evident in the data suggests that such events are unavoidable. If so, the task in dealing with extreme volatility should be to identify those technical or fundamental aspects of markets that make them precarious, rather than to attempt to eradicate catastrophes through regulation.

Footnotes

1. Log-relative price returns are also non-normally distributed. Close-to-close returns, for example, exhibit chi-squared statistics of 103.3 (d.f. = 22) for bonds, 94.4 (d.f. = 16) for notes and 337.3 (d.f. = 24) for Eurodollars.
2. Nonoverlapping weekly and monthly price change observations were also tested for normality. For the weekly data, normality was rejected at the 0.5 level of significance for bonds ($X^2 = 20.2$, d.f. = 11), notes ($X^2 = 19.6$, d.f. = 9) and Eurodollars ($X^2 = 21.4$, d.f. = 11). There were not enough observations to conduct meaningful tests of the monthly data.
3. Commodity Systems, Inc., 200 W. Palmetto Park Road, Boca Raton, Florida 33432.
4. See, for example, D. Hsu, R.B. Miller and D.W. Wichern, "On the Stable Paretian Behavior of Stock Market Prices," *Journal of the American Statistical Association* 69(345), pp. 108–113, and D.E. Upton and D.S. Shannon, "The Stable Paretian Distribution, Subordinated Stochastic Processes, and Asymptotic Lognormality: An Empirical Investigation," *Journal of Finance* 34, No. 4, pp. 1031–1039.
5. See, for example, E.C. Zeeman, *Catastrophe Theory: Selected Papers* (Reading, MA: Addison-Wesley, 1978), especially ch. 11, "On the Unstable Behavior of Stock Exchanges," and R. Savit, "When Random is Not Random: An Introduction to Chaos in Market Prices," *Journal of Futures Markets* 8, No. 3, pp 271–290.