Demystifying Illiquid Assets: Expected Returns for Private Equity

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ABSTRACT: The growing interest in private equity means that allocators must carefully evaluate its risk and return. The challenge is that modeling private equity is not straightforward, due to a lack of good quality data and artificially smooth returns. We try to demystify the subject, considering theoretical arguments, historical average returns, and a forward-looking analysis. For institutional investors trying to calibrate their asset allocation decisions for private equity, we lay out a framework for expected returns, albeit one hampered by data limitations, that is based on a discounted cash-flow framework similar to what we use for public stocks and bonds. In particular, we attempt to assess private equity’s realized and estimated expected return edges over lower-cost public equity counterparts. Our estimates display a decreasing trend over time, which does not seem to have slowed the institutional demand for private equity. We conjecture that this is due to investors’ preference for the return-smoothing properties of illiquid assets in general.

TOPIC: Private equity

KEY FINDINGS

- A leveraged small-cap public equity index may be a better benchmark for the performance of private equity than the large-cap indices generally used. Further, internal-rates-of-return (IRRs) can be especially misleading if they are compared against the time-weighted returns used for public market indices.
- The smoothed returns of private equity understate the true economic risk and are an artifact of the lack of mark-to-market for illiquid assets.
- The richening valuations of PE may be a headwind for future returns for the asset class, suggesting a slimmer edge over public equity than long-term averages.

As investors increasingly embrace private equity (PE), they find themselves posing the following questions: How much should they allocate? What are good yardsticks for assessing performance? Are the higher fees of PE justified by higher expected returns over public equity counterparts? What is the risk and diversification potential of PE?

The comparison to public equity is not straightforward. In general, illiquid assets are inherently harder to model, and this is exacerbated by a lack of good quality and transparent data. We try to demystify the subject of PE risk and return, focusing on the medium-term (5 to 10 years) expected return (ER) of PE. We view the topic through multiple lenses: theoretical required returns, historical performance, and, finally, our favored approach of extending our discounted-cash-flow-based methodology for equity and fixed income to the realm of PE. A common
framework helps highlight how the ER of PE is anchored to that of public equity by similar drivers, such as yield and growth. Although we focus on returns, our analysis also touches on the hidden risks and factor exposures of PE, and thereby suggests potentially better performance benchmarks and comparisons to public equity.

We observe that PE has grown in popularity despite a decreasing expected and realized return edge over public equity counterparts. We posit that this surprising outcome reflects the illusion of the lower risk of illiquid assets or the appeal of their artificially smooth return streams. Due to the absence of mark-to-market accounting, the reported volatility and equity beta of private assets tend to be understated, unless one desmooths their returns, which may not be a clear-cut exercise. This overstates their diversification potential or naively measured alpha. Even if one expected PE to provide zero excess return over public equity, the assumption that PE was less risky, and lowly correlated to public equity, would call for an increased allocation to PE. Furthermore, understating the reported risk compared to economic risk may, in itself, appeal to investors.

The shrinking valuation gap between private and public equity, which we show later on, is one indication of investor willingness to pay, perhaps knowingly overpay, for these return-smoothing characteristics.

This report is targeted at investors interested in understanding the relation between private and public equity at some depth, and it serves as a background piece for the readers of our annual Capital Market Assumptions edition of AQR Alternative Thinking. Our 2019 edition will now include a brief section on illiquid asset classes, which may suffice for most readers, but those wanting more detail can refer to this report.

FRAMEWORKS FOR EXPECTED RETURNS

We try to estimate the medium-term real ER for private equity, focusing on its edge over public equity. Specifically, our estimates are for the largest segment of the private equity market, US buyouts. Our expected return estimates are net of fees, as fees can be a substantial component of returns for illiquid assets.

We approach the topic through three complementary perspectives as described in Ilmanen (2011): theoretical required returns, historical evidence on past average returns, and yield-based analysis that considers current valuations and market conditions.

As is the norm with other asset classes, we present real (inflation-adjusted) compound rates of return for the asset class as a whole for a horizon of 5 to 10 years. Over such intermediate horizons, initial market yields and valuations tend to be the most important inputs. For multidecade forecast horizons, the impact of starting yields is diluted, so theory and long-term historical average returns (or yields) may matter more in judging expected returns. Our estimates are intended to assist investors with their strategic allocation and planning decisions, and, in particular, with setting appropriate medium-term expectations. They are highly uncertain and not intended for market timing. The broad framework may be more useful and informative than the point estimates themselves.

Theory

Theoretical or risk-based explanations of asset returns follow the premise that higher return is the compensation investors require for taking on additional risk. So, if PE has greater exposure than public equity to certain risk factors, and if those risk factors have positive expected returns, one would expect it to have both higher risk and return. Based on economic intuition and empirical evidence, which we describe later, we expect PE to have the following factor tilts over public equities: equity risk, (il)liquidity premium, size, and value:

1If PE is deemed to have an equity beta higher (lower) than 1, its alpha will be estimated to be lower (higher) than its simple excess return over public equities. For example, if one expects PE to deliver a 3% excess return over public equity and the equity risk premium to be 5%, the implied alpha of PE is 5% if PE’s beta to public equity is deemed to be 1.5, and 5.5% if using a beta of 0.5 based on the artificially smooth PE return series.

2A companion piece, Ilmanen et al. (2019), discusses another illiquid asset class, real estate.

3The terminology is mixed, as private equity could either refer to buyouts only or include other segments, like venture capital. We use the term to refer to buyouts only. Buyouts account for over 60% of the aggregate private equity market, as per market segment estimates by Døskeland and Strömberg (2018), based on Preqin data on funds raised 2012–2017.

4Gross expected returns could be 5% to 7% higher. See Døskeland and Strömberg (2018).
**Equity Risk:** The principles of corporate finance dictate that all else equal, companies with greater debt-to-equity (D/E) should have higher volatility and equity beta, as the required interest payment to debt holders increases the riskiness of the remaining cash flow to equity holders. Studies indicate that PE firms take on 100%–200% debt for every dollar of equity (down from the 300%–400% D/E ratios in the 1980s), whereas publicly listed firms, on average, add 50% of debt for every dollar of equity. This suggests PE’s equity beta is well above 1.6

Why, then, is PE vaunted for its diversification benefits? The answer lies in the lack of regular, mark-to-market pricing for illiquid assets in general. This induces the common practice of appraisal-based or self-reported NAVs that do not reflect the daily fluctuations in public markets, making for artificially smoothed returns that understate risk and correlation to public markets. Welch (2017) showed how traditional methods of valuation and financial reporting standards for PE understate its systematic risk and beta. Naïve regression-based equity beta estimates tend to be below 1.0, even if adjusting for illiquidity by using lagged betas.7 Many empirical studies have concluded that a beta estimate of 1.2–1.5 is more realistic, implying PE has higher volatility and lower risk-adjusted returns than naïve reported returns suggest.8

In short, although PE has low reported risk, it is economically riskier and has higher exposure to the equity risk premium than public equities, a combination that many investors may find appealing. The full risk of PE is most likely to materialize in prolonged bear markets, not in relatively fast ones like 2008–2009.

- **(II)liquidity Premium:** In principle, locking up capital for a 5–10 year window warrants a significant illiquidity premium, as suggested by Ang (2014).9 However, as we argue later, the data suggests that even if such a fair illiquidity premium existed, it may, in practice, be largely offset by investor willingness to overpay for the return-smoothing described earlier. Although this preference may be driven by end-investors, it may be exacerbated by agency issues. A fear of losing end-investor mandates may incentivize institutional investors (the limited partners in PE) to reduce the more readily apparent, reported short-term risks instead of latent, longer-term economic risks. As described in Welch (2017), investors averse to the price swings that come with regular trading may be content with net-of-fee PE returns that are merely on par with public equity, instead of setting the bar higher at returns that adequately compensate them for the higher equity beta and illiquidity of PE. The limited partners may also unknowingly overpay if they underestimate the equity beta of PE, making them attribute the higher returns of PE to an illiquidity premium or alpha, instead of the equity risk premium. Empirical evidence in private real estate supports our hypothesis. Over the long-run, private real estate has delivered returns below or on-par with publicly traded real estate investment trusts (REITs), instead of providing an illiquidity premium.10 If increased investor demand for PE drives

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5See Døskeland and Strømberg (2018). Further, Axelson et al. (2013) found median debt-to-enterprise value ratios of 70% for LBOs and 35% for public industry-and-region-matched companies, implying D/E ratios of 233% for LBOs and 54% for the public match. As of June 30, 2018, Bloomberg estimates of the weighted average D/E ratio for the S&P 500 and Russell 2000 are in-line, at 45% and 49%, respectively.

6As per Modigliani and Miller (1958), in general, \( r_l = r_u + (D/E) (r_e - r_d) (1 - T_c) \) where \( r_l \) = required rate of return on levered equity, \( r_u \) = unlevered cost of equity (or the return on assets) for an all-equity-financed firm, \( r_d \) = cost of debt, and \( T_c \) is the tax rate. This implies \( \beta_l = \beta_u + (D/E) (\beta_d - \beta_u) (1 - T_c) \). Thus, a levered firm (a firm with debt) has a higher required equity return and expected equity beta than an unlevered firm if \( D > 0 \) and \( r_d < r_u \).

7Desmoothing illiquid asset returns using a simple autoregressive AR1 variant may not suffice. Anson (2017) found that PE lagged betas are significant up to three quarters back; real estate lagged betas are significant up to four quarters back.

8See Døskeland and Strømberg (2018), which summarizes betas and risk factor loadings across several papers, datasets, and methodologies. All else equal, higher leverage tends to increase beta and performance fees dampen net-of-fee returns above the hurdle rate, thus lowering beta. See also Ang et al. (2017) and Phalippou (2014) for estimates of market beta and other factor tilts.

9Ang (2014) discussed a model that suggests that investors should require a 4%–6% illiquidity premium to lock up their capital for 5–10 years. However, broad evidence on realized illiquidity premia in many asset classes is mixed.

10See Ilmanen, Chandra, and McQuinn (2019) and Ang, Nabar, and Wald (2013). In North America, REITs have outperformed private real estate since inception, suggesting an inverse illiquidity premium. After adjusting for leverage and sector, the inverse premium almost disappears, but we still see no positive illiquidity premium.
up the purchase multiples of buyout targets, one may expect their future returns to be lower.

- **Size:** Buyout targets tend to have smaller capitalizations and, therefore, provide exposure to the size factor.\(^{11}\) This implies that a more appropriate benchmark would be a leveraged small-cap index that accounts for both the higher leverage and small-cap bias. The small-cap tilt is evident from holdings-based analyses that present the firm characteristics of typical buyout targets, but even this basic fact is hard to confirm with returns-based regressions, due to the artificial smoothness of PE returns.\(^{12}\)

- **Value:** Over and above a small-cap bias, buyout targets have tended to trade at lower valuation multiples than the market, though venture capital targets are more likely to be growth companies. Although Stafford (2017) and Chingono and Rasmussen (2015) reported a value tilt, broader evidence is mixed. Further, as we show later, the PE industry, overall, no longer has the valuation discount versus public equities it used to have. Although this may be partly due to a changing industry composition of buyout targets, it is unclear whether PE’s historical value bias will persist.

Crudely assuming that the fair illiquidity premium of PE gets fully offset by the overpayment for smooth returns, investors may still require and expect a higher return from PE than public equity, due to its higher equity beta and small-cap bias. This could thus help inform a public proxy approach for a minimum required return for PE. As a purely illustrative example, if investors assumed PE had a 1.2 beta and no net-of-fee alpha to public small-cap equities, and expected small-caps to return 5% excess of cash (this includes both the equity risk premium and the size premium), the implied PE net-of-fee expected excess return over cash would be 1.2 times 5%, or 6%.

**Historical Performance**

Now that we’ve outlined the economic rationale for the expected excess return of PE over public equities, what does historical evidence have to say?\(^{13}\) Exhibit 1 compares PE’s historical performance to various publicly traded benchmarks as well as to baskets of stocks that account for PE’s factor exposures.

For the reasons mentioned earlier, comparing PE reported returns directly to large-cap equities is not a fair measure of alpha or the illiquidity premium. Leveraged, small-cap indices are more appropriate as benchmarks. Exhibit 1 shows that over the period 1986 to 2017, PE outperformed large-caps by 2.3%, looking at arithmetic means (AM). But when compared to a 1.2\(\times\) leveraged small-cap index, this falls to just 0.7%, and PE actually underperformed a basket of small-cap value stocks by 1.6%.\(^{14}\) This is corroborated by Stafford (2017), who found that the long-run average excess returns of PE over public equity can be matched by a leveraged, small-cap value strategy. Thus, it appears that the PE industry, on average, has offered scant illiquidity premium beyond these typical factor tilts.\(^{15}\)

Nevertheless, for many investors, the bottom line is that PE firms have delivered clearly higher net-of-fee returns than the S&P 500 over the past 30 years, even if those excess returns could be largely accounted for by using more representative publicly traded benchmarks. Further, top-quartile managers would have served end-investors (LPs) better than the industry average results.\(^{16}\)

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\(^{11}\)Banz (1981) showed that, empirically, small-cap stocks have earned higher returns than large-cap stocks. The size premium is much debated and may not be as robust as other factor premia, as discussed in Alquist et al. (2018).

\(^{12}\)See L’Her et al. (2016), who found that the average size of LBOs is very small in comparison to listed small-cap equities.

\(^{13}\)We stress that the question of how much excess return over public equities investors require from PE (to compensate them for the greater illiquidity and risks associated with PE) is distinct from the question of how PE firms can generate those excess returns, over and above covering their high fees. The various ways PE firms meet this high return hurdle are described in Kaplan and Strömberg (2009) and Døskeland and Strömberg (2018). PE firms may be able to add significant value through prudent selection of buyout targets; opportunistic timing; as well as operational, financial, and governance engineering that improve the efficiency and growth prospects of the companies they hold.

\(^{14}\)The 1.2\(\times\) reflects the market beta of PE versus a small-cap index. Both Driessen et al. (2012) and Franzoni et al. (2012) found a beta of 1.3 against a broad stock index, implying a beta of 1.2 versus a small-cap index. Thus this 1.2\(\times\) small-cap proxy can also be viewed as the normative PE required return based on PE’s exposure to the equity premium and the size factor, or the PE expected return using a public proxy approach.

\(^{15}\)PE would look better if using geometric means (GMs) or public equity returns net of trading costs. In any case, the PE index returns in Exhibit 1 are IRRs that are not directly comparable to equity total returns.

\(^{16}\)That said, it may be too common for end-investors to assume that they can get top-quartile managers (which may be yet
PE managers’ exceptional skill becomes even more evident when we consider their performance before fees, which are estimated at around 6% per year.17

In Exhibit 1, we show internal-rate-of-return (IRR)-based returns for PE because they are commonly used. But we caveat that IRRs for individual managers are notorious for their gameability.18 A better metric of relative performance is the public market equivalent (PME) that is strongly preferred by academics (see Kaplan and Schoar 2005). The PME approach involves comparing another reason for the popularity of PE). Such overconfidence may be boosted by the fact that, as described by Harris et al. (2012), about half the universe of PE funds describe themselves as top quartile. The flexibility that PE funds have in slicing and dicing the data—comparison universes and time periods—makes this feat possible.

17 See Phalippou and Gottschalg (2009), who estimated PE fees to be 6% per year. It is not straightforward to translate typical PE fund fees of 2% management fee, 20% carry, a hurdle rate, and additional portfolio company fees into a fixed yearly fee. Døskeland and Strömberg (2018) cited a McKinsey (2017) CEM Benchmarking study among large institutional investors, which estimates total fees to be 5.7% p.a. comprising 2.7% in management fees, 1.9% in carried interest (performance fee), and 1.2% for other fees, including net portfolio company fees.

18 Unlike most index returns, IRRs are not time-weighted and are affected both by the magnitude and timing of cash flows. Larger cash flows have a greater effect on IRRs, and IRR calculations embed a noninnocuous assumption that interim cash flows can be reinvested at the IRR. Thus, PE GPs can time capital calls from LPs, as well as deal exits so as to boost IRRs.

the amount of capital generated by a PE strategy to that generated by a public market index (the benchmark) over the lifespan of the fund, assuming similar amounts were invested with the same timing. For example, Harris et al. (2014) found a long-run average PME of roughly 1.2 versus the S&P 500, which is in line with Kaplan and Sensoy (2015). A PME of 1.2 against the S&P 500 implies 20% outperformance by PE over the period capital is deployed. Assuming a typical investment period of six years, that implies PE has outperformed the S&P 500 by 3.1% annually, net-of-fees.

Exhibit 2 plots PMEs for various vintages, alongside the valuation gap at deal inception between public equity and PE.19 Panel A plots the EBITDA/EV, or inverted purchase multiple, for PE alongside the EBITDA/EV for public equity, and thus depicts the ex-ante return edge PE may have due to its lower valuations. Panel B plots PMEs for each vintage-year, with and without adjustments for leverage, size, and sector, and thus shows the future realized return edge of PE over two public equity benchmarks (roughly for the next five to six years). We see that as PE has grown relatively richer and the valuation gap has

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**EXHIBIT 1**

**Historical Performance of Private Equity: Scant Illiquidity Premium July 1, 1986–December 31, 2017**

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<td><strong>Average return</strong></td>
<td>9.9%</td>
<td>9.9%</td>
<td>7.5%</td>
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<td><strong>(Arithmetic)</strong></td>
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<td><strong>Excess return over</strong></td>
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<td>2.3%</td>
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<td><strong>public (Arithmetic)</strong></td>
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<td><strong>Average return</strong></td>
<td>9.8%</td>
<td>9.2%</td>
<td>6.4%</td>
<td>5.5%</td>
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<td>6.7%</td>
<td>9.4%</td>
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<td><strong>(Geometric)</strong></td>
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<td><strong>Excess return over</strong></td>
<td>3.4%</td>
<td>4.3%</td>
<td>3.7%</td>
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<td>0.4%</td>
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<td><strong>public (Geometric)</strong></td>
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<tr>
<td><strong>Volatility</strong></td>
<td>13.8%</td>
<td></td>
<td>15.8%</td>
<td>20.7%</td>
<td>24.9%</td>
<td>19.4%</td>
<td>21.4%</td>
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Notes: PE returns are based on pooled horizon IRRs, net of fees, expenses, and carried interest. Public index returns are gross of fees and of trading costs. Excess return over public refers to the raw Cambridge PE return in excess of each of the public market indices to the right.

Sources: AQR, Bloomberg, Cambridge Associates (using internal-rate-of-return [IRR]-based raw index returns and an AR[1]-desmoothed variant), Kenneth French Data Library.

For illustrative purposes only and not representative of any portfolio or strategy that AQR currently manages.
narrowed, PE’s outperformance over public equities has declined, with post-2006 vintages realizing lower outperformance than prior vintages. This is in line with the empirical findings of other studies. Harris et al. (2014, 2016) observed PMEs near 1 after 2006, implying that PE has had no edge over public equity since 2006. This claim is tempered by Brown and Kaplan (2019) who use updated data on US vintages from 2009–2014 and find PMEs above 1 even for vintages after 2006. In either case, however, PE PMEs are clearly lower after 2006. Industry participants, too, like Bain (2019), have noted that buyout returns appear to be not as robust as they were before the global financial crisis.

What could possibly explain the post-2006 decline in PE’s realized excess returns? We surmise that this may have been caused by the relative richening of PE as observed in Exhibit 2, and a gradual decline in PE leverage (D/E).²⁰

²⁰In Appendix A, we explore the declining trend in expected returns through our yield-based framework, which we introduce in the next section. Although this attribution is done on expected returns, it may shed light on the downward trend in realized returns, as well.
The increasing appetite for PE may have spurred the rising valuations of PE, though shifting sector weights may have played a part, too. Investors may have been encouraged by PE’s strong track record versus public equity in the early 2000s, and broad dissemination of the endowment model espoused by Yale’s David Swensen. Later on, the issuance of FAS 158 in 2006 and the shallower reported drawdowns of PE during the global financial crisis may have accentuated this trend. Today, industry reports like Bain (2019) report a record-high amount of dry powder (capital committed to PE that is yet to be invested) and greater competition for acquisitions from cash-rich corporations, suggesting that demand for PE has not waned.

Simultaneously, regulation enacted in the wake of the financial crisis has curtailed the amount of debt that companies can take, potentially reducing the returns from financial leverage. This may have been partly offset by lower financing rates in the last decade, but Axelson et al. (2013) suggested that when access to credit is easier, acquirers may bid up purchase prices, leading to lower future returns for the vintage. Still, any future relaxation of regulatory requirements could promote greater debt financing that may boost PE returns.

The key question, then, is what net returns end-investors can expect in today’s environment of tighter valuations and greater competition for deals. How should we weigh the longer 30-year history and the more recent 10-year evidence when estimating future PE outperformance?

Normally we give greater credence to longer-run evidence, but two disconcerting trends point to overestimating the more recent history. First, Exhibit 2 depicts a shrinking valuation gap between PE and public equities, and PE outperformance ceased at roughly the same time as the valuation gap closed. This corroborates our earlier point that increasing investor demand may have driven up PE valuations. Second, many academic studies show that PE fund returns tend to be lower after hot-vintage years characterized by high fundraising activity or capital deployment, attractive financing conditions, and easy leverage. Skeptics stress that the current environment can be characterized by low financing rates coupled with increasing institutional demand for PE, more PE firms, record-high dry powder, and competition from cash-rich public companies and sovereign wealth funds. Thus, PE faces headwinds that make it less likely to deliver the strong returns it has in the past. Of course, richness versus history is not unique to PE: As described in our Capital Market Assumptions editions of AQR Alternative Thinking, many other asset classes appear expensive today, perhaps reflecting the easy global monetary policies of the 2010s. If public equities were to perform poorly, as in 2018, PE PMEs could still end up relatively higher.

In contrast to our conservative forecasts, institutional investors widely expect PE to outperform public equity by 2%–3%. Despite its recent lack of outperformance, investors remain optimistic on PE, even as they increasingly question the value-add of other forms of active management. Some reasons for this may be the lack of transparency on PE returns and fees, slow learning about performance, and the use of misspecified benchmarks. PE returns are often presented as IRRs, which can be easy to game and which evolve slowly. It’s also plausible that investors are cognizant of the points we raise and knowingly accept a more modest, even zero, net-of-fee outperformance over public equities because they find the artificially smoothed returns of private assets desirable.

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21 The NACUBO Endowment study is conducted annually on around 800 US colleges and universities, and reports an increase in endowment allocation to private equity from 0.9% (3.2%) in 2002 to 5% (10%) in 2018 on an equal-weighted (dollar-weighted) basis.

22 FAS 158 required employers to report the overfunded or underfunded status of a defined benefit post-retirement plan on their balance sheets. This may have made some institutional investors more sensitive to mark-to-market fluctuations.

23 The Leveraged Lending Guidance introduced in 2013 recommended capping pro-forma leverage (total debt to pro-forma EBITDA) for leveraged financing at 6X and also required that a company be able to amortize at least 50% of its debt within five to seven years of closing. Subsequently, banks and sponsors often found themselves unable to execute deals that exceeded that threshold as that invited greater regulatory scrutiny and possible penalties. More recently, the regulatory stance has softened to emphasize flexibility around the previously issued guidelines.

24 Kaplan and Strömberg (2009), Axelson et al. (2013), Harris et al. (2014), Robinson and Sensoy (2016), and L’Her et al. (2016) found that private equity returns are inversely related to the amount of money flowing into the PE industry, as well as GP access to cheap financing.

25 Andonov and Rauh (2018) found that institutional investors extrapolate past performance when setting return expectations, and in recent years have expected the PE industry to outperform public equity by 2.5% (arithmetic) and 1.5% (geometric) over the long run.
Finally, irrespective of whether one uses IRRs or PMEs, the choice of benchmark is critical. Exhibit 2 Panel B reveals that PE PMEs for all vintages are lower if using a $1.2 \times$ small-cap index instead of the S&P 500, and even drop below 1, implying underperformance against public equity once PE’s small-cap bias, sector tilts, and higher leverage are accounted for. The right benchmark may be debatable, but one key consideration is whether the benchmark accounts for the market and factor exposures of PE that can be obtained passively.

**Yield-Based Approach**

Our third approach to PE ER estimation is yield-based. Here we apply the discounted cash flow framework we use to forecast 5–10 year expected returns of public equities and bonds in our Capital Market Assumptions editions of AQR Alternative Thinking. Admittedly, each of our inputs is debatable as data limitations on PE necessitate many simplifying assumptions. Still, the broad framework remains relevant, possibly more so than the point estimates, as it explains the mechanism of how PE firms can generate higher returns than public equity. PE firms can employ multiple levers to boost returns: namely, higher starting yields through deal selection; higher earnings growth rates through operational improvements; multiple expansion through opportunistic timing of entries/exits; and financial leverage. We should expect yields and growth rates for PE to be at least loosely anchored to those for public equities.

Exhibit 3 illustrates our framework for PE ER. First, we estimate unlevered ER $r_u$ using the dividend discount model: $r_u = y_u + g_a$, where $y_u$ = dividend yield and $g_a$ = real earnings-per-share growth rate. Then, we estimate the theoretical required levered return to equity $r_l$ by plugging in leverage $D/E$ and the cost of debt $k_d$, to which we finally add expected multiple expansion $m$ to arrive at gross PE ER $r_g$.

We describe the framework and our assumptions here:

- **Yield:** Although PE does not have regular payouts the way public equities pay dividends, we assume the income yield of PE to be half of its expected dividend yield.

Sources: AQR, Pitchbook, Bain & Company (2018), Bloomberg, CEM Benchmarking, Consensus Economics. Current estimate as of September 30, 2018, and subject to change. Historical averages cover period January 1, 1993 to September 30, 2018. There is no guarantee, express or implied, that long-term return targets will be achieved. Realized returns may come in higher or lower than expected. For illustrative purposes only and not representative of any portfolio or strategy that AQR currently manages.
unlevered earnings yield, along the lines of our methodology for public equities.27 We define PE’s unlevered earnings yield as its EBIT-to-EV, which we somewhat crudely estimate as half of its EBITDA-to-EV, based on historical averages observed for public equities.28 We average EV/EBITDA purchase multiples from several sources, as data on PE purchase multiples often covers only a small subset of the deal universe and can thus be noisy. As seen in Exhibit 2, until the mid-2000s, PE yields were almost always higher than public equity yields.

- **Growth Rate:** We assume an unlevered real growth rate of 3%, which is more than double what we assume for public equities. Thus, we assume that, even in the absence of financial leverage, PE firms may achieve higher growth rates through operational improvements resulting in higher margins, and by being overweight sectors with higher growth rates.29 Leverage further amplifies this effect if operating income exceeds the interest expense. Our 3% unlevered real growth assumption factors in both the initial higher growth rate as the GPs improve operations, as well as the later, lower steady-state growth rate after the company goes public. The latter will be closer to our 1.5% earnings-per-share real growth assumption for public equities.

27 See AQR Alternative Thinking Q1 2017: Capital Market Assumptions. Historically, the dividend payout ratio for public equities (the S&P 500) has averaged roughly 50% over the period January 1, 1900 to December 31, 2016.

28 For the Russell 2000, on average, EBITDA is roughly twice EBIT over the period January 1, 1995 to September 30, 2018. We perform this additional step of estimating EBIT-to-EV from EBITDA-to-EV, as EBIT-to-EV does not include depreciation and amortization and is thus more comparable to the earnings yield we use for public equities, that is based on net income and thus, net of depreciation and amortization too.

29 Acharya et al. (2013) found that PE ownership causes the operating margin (EBITDA/Sales) to increase by around 4% on average relative to the pre-acquisition phase; Guo et al. (2011) reported an even higher 12% increase in net cash flow to sales. On the other hand, Cohn et al. (2014) suggested that operating improvements are way more modest and emanate from a natural mean-reversion in operating efficiency, not the changes introduced by PE GPs. Additionally, Døskeland and Strömberg (2018) found that PE tends to be overweight technology and underweight financials, even if excluding the venture capital and growth equity segments of PE.

- **Leverage:** For the post-2008 period, we interpolate annual D/E ratios from Pitchbook/Bain. Pre-2008, we assume an aggregate D/E ratio that tapers from 300% in the 1990s to the 150% D/E reported in 2008. Although historical simulations often assume a constant D/E ratio of, say, 200% over time, PE leverage levels have been trending lower despite lower funding costs.30

In principle, higher leverage (D/E) should boost equity returns if operating income is greater than the interest expense on the debt. In reality, however, Axelson et al. (2013) showed that low funding costs and high leverage tend to coincide with hot, overpriced vintages that have lower future returns. Our usage of PE yield and multiple expansion partly captures this effect.

- **Cost of Debt:** We estimate PE’s cost of debt as real LIBOR plus a spread proxied by 33% of the high-yield (HY) index OAS over duration-matched Treasuries. Studies indicate PE firms have superior access to credit and borrow more when credit is cheap. A high proportion of PE debt is secured bank debt financed at floating rates plus a spread. In the absence of historically accurate bank loan data for PE, we proxy the PE bank loan spread as two-thirds of the OAS of the HY index over duration-matched Treasuries.31 As the entire spread overstates the cost of debt ultimately borne by the firm, we estimate the actual PE cost of debt as half of this, at real LIBOR plus one-third of the HY OAS.32 As the purchase multiples we use are

30 Both Axelson et al. (2013) and L’Her et al. (2016) found a decreasing trend in D/E for LBOs. This likely reflects evolving risk preferences by GPs (bigger firms protecting their brands) and their LP clients (pension funds may be less risk tolerant than family offices and endowments).

31 Axelson et al. (2013) suggested that a large proportion of LBO debt is secured bank debt that is generally financed at lower rates than HY bonds. Demiroglu and James (2010) found that over the period 1997 to 2007, PE firms incurred a spread over LIBOR that averaged around 70% of the HY OAS and was less sensitive to credit conditions than HY spreads. Our use of a constant proportion of the HY OAS may thus overstate the PE cost of debt when HY spreads spike, as in 2008-9.

32 As explained in AQR Alternative Thinking Q1 2016, investors do not earn the entire credit spread as they incur default losses. Giesecke et al. (2011) found that over the long term, the average credit risk premium that investors realize is roughly half the average credit spread. The flip side of this is that the issuing firm’s
one-year averages, we also use one-year averages of the cost of debt.

The tax deductibility of interest expense decreases a company’s de facto cost of debt, giving rise to a debt tax shield that is touted as a value-add of PE. However, as tax laws vary by jurisdiction, we do not account for taxes here. Our assumption effectively increases the cost of debt for PE, but this is offset by the aforementioned haircut we apply to the cost of debt.

**Multiple Expansion:** The principle of mean-reversion suggests that PE multiple expansion is more likely if it has an initial discount versus the market. Hence, we estimate the return from multiple expansion as the annualized return if PE multiples converged partly, say 20% of the way, toward the initial public market multiple, over the lifetime of the deal, assumed to be six years. We floor this return at zero; that is, we do not allow for multiple compression, as there is evidence that PE GPs delay exits so as to sell at higher multiples.

We assume no multiple expansion in our yield-based frameworks for passively managed public equities and fixed income. However, we make an exception for PE due to its active ownership and some evidence that PE GPs can time deal entries and exits. Given the arbitrary nature of our estimate, our general skepticism around multiple expansion for any asset class, and the noise in data on PE purchase multiples, we choose to apply only a conservative, partial convergence towards the market multiple.34

**Fees:** We assume PE fees of 5.7%, as per the McKinsey (2017) CEM Benchmarking survey estimate average. PE fees are partly based on performance. As we expect future PE returns to be lower than in the past and our assumption of 5.7% is based on historical averages, the performance component of fees may be lower going forward. Nevertheless, we stick with the historical average of 5.7% as our best estimate, as PE fees can vary vastly based on deal terms.

Putting it together, we estimate a real ER for US PE of 9.6% gross and 3.9% net-of-fee.35 In comparison, our US public equity real return estimate is 3.1%, net of a 10bps fee for passive investing.36 We thus expect PE to have a roughly 80bps higher net-of-fee ER.37 As mentioned earlier, we do not interpret this outperformance of PE as an illiquidity premium, but the warranted risk premium given the higher equity risk of PE.

Our current estimate of PE outperformance is undoubtedly low compared to history. Exhibit 4 charts net-of-fee PE ER and public equity ER through time (we caveat that limited data especially in the earlier part of the sample necessitates the use of simplifying assumptions). The multiple expansion we assume here does not involve assuming that the whole PE asset class richens but rather that even amidst an unchanged capital market environment, at the time of deal exit, individual deals are able to justify higher multiples than their purchase multiples, thanks to GPs’ skills in entry/exit timing or to operational improvements that boost expected growth beyond the going-public date. This is analogous to the roll-down gains bonds can earn when we assume an unchanged yield curve. The market timing skills of GPs may not help LPs if they fund their PE allocations from public equity.

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35 PE-realized returns and ERs can be measured for a given vintage or across many vintages for a given period. Our approach uses the most recent purchase multiple and tries to loosely estimate the ER for the current vintage year for the next 5–10 year period. (In practice, vintage year data may provide better transparency on prevailing PE valuations even if it only reflects just-deployed capital. We implicitly assume that if the capital deployed in previous vintages were properly marked-to-market at the same point in time, and was deployed to a similar mix of industries, under similar financing conditions, it would have similar valuations and expected returns. In this light, our estimates apply to the whole buyout market, although a value-weighted purchase multiple may better represent the entire market than the median purchase multiple we use.)
36 See AQR Alternative Thinking Q1 2017. Averaging our two methods for public equities would lead to a higher ER for US public equity (the S&P 500), as that includes net buybacks. However, here we estimate public equity real ER using only the earnings yield based methodology, ignoring net buybacks, as that is closer to our PE framework.
37 It may seem misleading to compare PE fees to equity index fund fees, when a more natural comparison would be against active public equity funds. Because our focus is on asset class expected returns, we use public equity index funds as the implicit PE benchmark, but we note that the PE edge over public equity would be higher against a benchmark of active equity funds if the latter are collectively assumed to underperform passive funds due to their higher fees.
The gyrations in the PE ER line are driven mostly by fluctuations in the cost of debt, as it is the only input based on mark-to-market data. We clearly discern a downtrend in PE ER from the 1990s to the 2010s, driven by richening PE multiples (resulting in both lower yields and lower multiple expansion) and a gradual decline in leverage. The early 1990s and 2002–2005 were halcyon years when both PE valuations and the cost of debt were low; it is no wonder then that those vintages delivered high subsequent returns. Our current outlook is far more modest reflecting PE's rich valuations and low leverage. The Appendix includes a visual decomposition of the PE expected return edge over public equity.

One limitation of our PE framework may be its sensitivity to the cost of debt, and our imperfect proxy for the cost of debt. During periods like 2000 when real cash rates were high, or 2008–2009 when credit spreads spiked, our cost of debt may be overstated and our methodology can give misleadingly low estimates of broad PE market ER. Under such conditions, new leveraged buyouts become uneconomical and primary PE markets slow down as the cost of debt is too high to warrant more leverage. On these rare occasions, the secondary market may be more active and provide a better estimate of actual transaction prices and thus the broad PE market ER. During the 2008–2009 turmoil, the secondary market pointed to a very high ER, which seems more intuitive than the low ER seen in Exhibit 4.

CONCLUSION

Our analysis suggests that private equity does not seem to offer as attractive a net-of-fee return edge over public market counterparts as it did 15–20 years ago, from either a historical or forward-looking perspective. Institutional interest in private equity has increased despite its mediocre performance in the past decade versus corresponding public markets, and weak evidence on the existence of an illiquidity premium. Although this demand may reflect a (possibly misplaced) conviction in the illiquidity premium, it may also be due to the appeal of the smoothed returns of illiquid assets in general.

It’s possible that investor overpayment for the smoothing characteristics offsets a large part of the fair illiquidity premium. Recent surveys suggest that investors still have high expectations of prospective PE returns. This may be due to the inherent difficulty of modeling illiquid assets, and lack of transparency on fees and performance. In this article, we present more comparable benchmarks or suitable adjustments for evaluating past performance, and a yield-based framework to estimate future returns. Although some specific assumptions are debatable, our framework helps to illustrate the basic arithmetic or the moving parts underlying expected returns for private equity.
We humbly admit that return estimates for any asset class come with a great deal of uncertainty, and our framework is a work in progress that we may fine-tune in the future. We hope it is a first step toward a more intuitive and transparent comparison between public and private equity.

**APPENDIX**

**DECOMPOSITION OF THE EXCESS RETURNS OF PRIVATE EQUITY OVER PUBLIC EQUITY**

In the main body, we report a decline in both the realized and estimated excess return of PE over public equities. To shed light on the possible drivers of this trend and to help readers understand our framework for expected returns, we decompose the expected PE net-of-fee return edge over public equity into its different moving parts. We describe the decomposition in the following:

As per AQR Alternative Thinking Q1 2018 and the dividend discount model,

\[
\text{Net-of-fee Public Equity } ER = y_{pub} + g_{pub} + m_{pub} - f_{pub} \tag{1}
\]

where
- \(y_{pub}\) = dividend or equity payout yield
- \(g_{pub}\) = long-term expected growth rate (assumed to be a constant 1.5%) 
- \(m_{pub}\) = multiple expansion (assumed to be zero) 
- \(f_{pub}\) = management fee (assumed to be 10 bps) for public equities

As per Exhibit 3 and the Modigliani–Miller equation,

\[
\text{Net-of-fee PE } ER = r_u + \frac{d}{d^u} \times (r - k_d) + m_{pet} - f_{pet} \tag{2}
\]

where
- \(r_u\) = unlevered PE ER
- \(\frac{d}{d^u}\) = debt-to-equity 
- \(k_d\) = PE cost of debt
- \(m_{pet}\) = PE multiple expansion
- \(f_{pet}\) = PE fees (assumed to be a constant 5.7%) for PE

As \(r_u = y_u + g_u\), Equation 2 can be rewritten as

\[
\text{Net-of-fee PE } ER = y_{pet} + g_{pet} - d_{pet} + m_{pet} - f_{pet} \tag{3}
\]

where
- \(y_{pet} = y_u \times (1 + \frac{d}{d^u})\), that is, the levered yield of PE

\[g_{pet} = g_u \times (1 + \frac{d}{d^u})\], that is, the levered growth rate of PE

\[d_{pet} = k_d \times \frac{d}{d^u}\], that is, the interest expense or payout to debtholders

Thus, subtracting (1) from (3), the PE net-of-fee return edge over public equity can be attributed to five components:

- **Levered yield differential**: This differential depends on both unlevered yields, as well as leverage ratios (D/E). The relative thinning of PE’s unlevered yield as depicted earlier in Exhibit 2 Panel A, coupled with decreasing PE leverage, has led to a gradual decline in this component.

- **Levered growth differential**: As we make the simplifying assumption of constant unlevered growth rates for both private and public equity, the difference in the levered growth rates is driven entirely by the time-varying leverage of PE. The near-steady decline we see is due to the declining trend in PE leverage.

- **Multiple expansion differential**: As we assume zero multiple expansion for public equities, this equals PE expected multiple expansion. As seen in Exhibit A1, this is just a small component of the return differential.

- **Fee differential**: As we assume constant fees for PE (5.7%) and public equities (10bps), this is a constant -5.6%.

- **PE payout to debtholders**: The PE payout to debtholders detracts from the returns to PE equity-holders. We observe a declining trend in this component due to a gradual decline in both rates and PE leverage, but that is more than offset by the decline in other components. Publicly listed firms also pay interest expense to their debt-holders. But as our method for public equity ER starts from dividend yields, it is already net of interest expense, and thus already accounted for.

Exhibit A1 plots these components and reveals that, under our yield-based framework, the declining expected return differential of PE over public equity has been driven first by the relative thinning of PE and second by the decrease in PE leverage which is reflected in both the declining levered growth differential and partly in the decreasing levered yield differential.\(^{40}\) As PE leverage has declined from around 300% D/E in the 1990s to 100%−150% D/E more recently, it has had less of an amplifying effect on PE ER.

\(^{40}\)For simplicity, the decomposition shows the difference in levered yields and levered growth rates. Thus, it does not disentangle the effect of time-variation in PE leverage from the effect of time-variation in the unlevered yield.

Penman et al. (2018) showed the relation between unlevered and levered earnings yield, as per Modigliani and Miller (1958).


**EXHIBIT A1**

Decomposition of the Net-of-Fee Expected Return Differential of Private Over Public Equity
January 1, 1993–September 30, 2018

![Graph showing the decomposition of the Net-of-Fee Expected Return Differential of Private Over Public Equity from 1993 to 2018.](image)


**REFERENCES**


Disclosure
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ADDITIONAL READING

Fact, Fiction, and the Size Effect
RON ALQUIST, RONEN ISRAEL, AND TOBIAS MOSKOWITZ
The Journal of Portfolio Management
https://jpm.pm-research.com/content/45/1/34

ABSTRACT: In the earliest days of empirical work in academic finance, the size effect was the first market anomaly to challenge the standard asset pricing model and prompt debates about market efficiency. The notion that small stocks have higher average returns than large stocks, even after risk adjustment, was a path-breaking discovery, and for decades it has been taken as an unwavering fact of financial markets. In practice, the discovery of the size effect fueled a crowd of small-cap indexes and active funds to the point that the investment landscape is now segmented into large and small stock universes. However, despite its long and illustrious history in academia and its commonplace acceptance in practice, there is still confusion and debate about the size effect. We examine many claims about the size effect and aim to clarify some of the misunderstanding surrounding it by performing simple tests using publicly available data. For one, using 90+ years of US data, there is no evidence of a pure size effect; moreover, it may not have existed in the first place, if not for data errors and insufficient adjustments for risk and liquidity.

Measuring Liquidity Premiums for Illiquid Assets
MARK ANSON
The Journal of Alternative Investments
https://jai.pm-research.com/content/20/2/39

ABSTRACT: Private assets, such as private equity, venture capital, and real estate, have long been a thorn in the side of asset allocators and chief investment officers. Their lack of liquidity makes it hard to analyze their return stream and to construct a performance attribution. The illiquid nature of these assets often leads to mis-specification and estimation of the systematic risk embedded in their returns and the true amount of alpha generated by these managers.

The Components of Private Equity Performance: Implications for Portfolio Choice
WILL KINLAW, MARK KRITZMAN, AND JASON MAO
The Journal of Alternative Investments
https://jai.pm-research.com/content/18/2/25

ABSTRACT: Kinlaw, Kritzman, and Mao use a proprietary database of private equity returns to measure the excess return of private equity over public equity and to partition this return into two components: an asset class alpha and compensation for illiquidity. Their evidence suggests that private equity managers, as a group, generate alpha by anticipating the relative performance of economic sectors. The authors assume that manager-specific alpha is fully diluted across a broad universe of private equity managers to interpret the balance of excess return as a premium for illiquidity. Their results suggest that investors can capture the asset class alpha of private equity by using liquid assets such as exchange-traded funds to match the sector weights of private equity investors. This decomposition of private equity performance has important implications for portfolio choice, which Kinlaw, Kritzman, and Mao explore in this article.